

THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL OF TANZANIA
ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

142/1

ADVANCED MATHEMATICS 1

(For Both School and Private Candidates)

Time : 3:20 Hours

ANSWERS

Year : 2000

Instructions

1. This paper consists of section A and B.
2. Answer ALL questions in section A any FOUR (4) questions from section B.
3. Communication devices and any unauthorised materials are **not** allowed in the examination room.
4. Write your **Examination Number** on every page of your answer booklet(s).

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2. (a) Find the equations of the tangents from the origin to the circle $x^2 + y^2 - 5x - 5y + 10 = 0$.

Equation of circle: $x^2 + y^2 - 5x - 5y + 10 = 0$

Complete squares:

$$x^2 - 5x + y^2 - 5y + 10 = 0$$

$$(x - 2.5)^2 - 6.25 + (y - 2.5)^2 - 6.25 + 10 = 0$$

$$(x - 2.5)^2 + (y - 2.5)^2 = 2.5$$

So, center (2.5, 2.5), radius $\sqrt{2.5}$.

Tangents from (0,0) have equation $y = mx$.

Substitute in circle:

$$(x - 2.5)^2 + (mx - 2.5)^2 = 2.5 \quad (1)$$

$$+ m^2)x^2 - 5(1 + m)x + 10 = 0$$

For tangency, discriminant = 0:

$$[-5(1 + m)]^2 - 4(1 + m^2)(10) = 0$$

$$25(1 + m)^2 - 40(1 + m^2) = 0$$

$$25 + 50m + 25m^2 - 40 - 40m^2 = 0$$

$$-15m^2 + 50m - 15 = 0$$

$$15m^2 - 50m + 15 = 0 \text{ Divide}$$

$$5: 3m^2 - 10m + 3 = 0$$

$$\text{Solve: } m = [10 \pm \sqrt{(100 -$$

$$36)] / 6$$

$$= [10 \pm \sqrt{64}] / 6 = [10 \pm 8] / 6$$

$$m = 18/6 = 3, \text{ or } m = 2/6 = 1/3$$

So tangents:

$$y = 3x, y = (1/3)x$$

2. (b) Find the shortest distance from the point $P(-1, 2)$ to the circle $x^2 + y^2 - 6x - 8y + 21 = 0$.

Circle: $x^2 + y^2 - 6x - 8y + 21 = 0$

Complete squares:

$$(x - 3)^2 - 9 + (y - 4)^2 - 16 + 21 = 0$$

$$(x - 3)^2 + (y - 4)^2 = 4$$

Center = (3,4), radius = 2.

Distance d from $P(-1,2)$ to center (3,4):

$$d = \sqrt{(-1 - 3)^2 + (2 - 4)^2} = \sqrt{(-4)^2 + (-2)^2}$$

$$= \sqrt{16 + 4} = \sqrt{20} = 2\sqrt{5}$$

$$\text{Shortest distance} = d - r = 2\sqrt{5} - 2$$

Answer: $2\sqrt{5} - 2$

3. (a) A and B are points (3,4) and (-1,2). Find the cartesian equation of the locus of $P(x,y)$ such that $AP = 2BP$.

$$AP^2 = 4BP^2$$

$$(x - 3)^2 + (y - 4)^2 = 4[(x + 1)^2 + (y - 2)^2]$$

Expand:

$$(x^2 - 6x + 9 + y^2 - 8y + 16) = 4(x^2 + 2x + 1 + y^2 - 4y + 4)$$

$$x^2 + y^2 - 6x - 8y + 25 = 4x^2 + 8x + 4 + 4y^2 - 16y + 16$$

$$\begin{aligned} \text{Bring all terms: } x^2 + y^2 - 6x - 8y + 25 - (4x^2 + 8x + \\ 4y^2 - 16y + 20) &= 0 \\ x^2 - 4x^2 + y^2 - 4y^2 - 6x - 8x - 8y + \\ 16y + 25 - 20 &= 0 \\ -3x^2 - 3y^2 - 14x + 8y + 5 &= 0 \end{aligned}$$

$$\text{Multiply } -1: 3x^2 + 3y^2 + 14x - 8y - 5 = 0$$

$$\text{Answer: } 3x^2 + 3y^2 + 14x - 8y - 5 = 0$$

4. Function $f(x) = (3x - 2)/(x + 1)$, domain $\mathbb{R} \setminus \{-1\}$.

$$(a) \text{ Range: } y = (3x - 2)/(x + 1) \quad yx + y = 3x - 2 \quad yx - 3x = -2 - y \quad x(y - 3) = -(y + 2) \quad x = -(y + 2)/(y - 3)$$

So defined $\forall y \neq 3$.

$$\text{Range} = \mathbb{R} \setminus \{3\}$$

(b) $f^{-1}(x)$:

$$\text{Swap } x, y: x = (3y - 2)/(y + 1)$$

$$x(y + 1) = 3y - 2 \quad xy + x = 3y$$

$$-2 \quad xy - 3y = -x - 2 \quad y(x - 3)$$

$$= -(x + 2) \quad y = -(x + 2)/(x - 3)$$

$$\text{So } f^{-1}(x) = -(x + 2)/(x - 3), \text{ domain } \mathbb{R} \setminus \{3\}.$$

$$(c) \text{ Domain of } f^{-1} = \mathbb{R} \setminus \{3\}, \text{ Range} = \mathbb{R} \setminus \{-1\}$$

5. (a) Evaluate

$$(i) \tan(\sin^{-1}(3/4))$$

Let $\theta = \sin^{-1}(3/4)$, so $\sin\theta = 3/4$. Then $\cos\theta$
 $= \sqrt{1 - (9/16)} = \sqrt{7/16} = \sqrt{7}/4$. $\tan\theta =$
 $\sin\theta/\cos\theta = (3/4) / (\sqrt{7}/4) = 3/\sqrt{7}$.

Answer: $3/\sqrt{7}$

(ii) $\cos(\sin^{-1}x)$

Let $\theta = \sin^{-1}x$, so $\sin\theta = x$. $\cos\theta$
 $= \sqrt{1 - x^2}$.

Answer: $\sqrt{1 - x^2}$

(b) Show $(\cot\theta + \csc\theta)^2 = (1 + \cos\theta)/(1 - \cos\theta)$.

$$\begin{aligned}\text{LHS} &= (\cot\theta + \csc\theta)^2 \\ &= (\cos\theta/\sin\theta + 1/\sin\theta)^2 \\ &= ((\cos\theta + 1)/\sin\theta)^2 \\ &= (1 + \cos\theta)^2/\sin^2\theta\end{aligned}$$

$$\text{But } \sin^2\theta = 1 - \cos^2\theta = (1 - \cos\theta)(1 + \cos\theta)$$

So $\text{LHS} = (1 + \cos\theta)/(1 - \cos\theta)$ **Proved.**

(c) If $8\sin^2\theta + 2\cos\theta - 5 = 0$, show $\cos\theta = 3/4$ or $-1/2$.

Use $\sin^2\theta = 1 - \cos^2\theta$:

$$8(1 - \cos^2\theta) + 2\cos\theta - 5 = 0$$

$$8 - 8\cos^2\theta + 2\cos\theta - 5 = 0$$

$$-8\cos^2\theta + 2\cos\theta + 3 = 0$$

$$8\cos^2\theta - 2\cos\theta - 3 = 0$$

$$\text{Solve: } \cos\theta = [2 \pm \sqrt{4 + 96}]/16 = [2 \pm 10]/16$$

$$= (12/16) = 3/4 \text{ or } (-8/16) = -1/2 \text{ **Proved.**}$$

Values of $\tan\theta$:

If $\cos\theta = 3/4$, then $\sin\theta = \pm\sqrt{1 - 9/16} = \pm\sqrt{7/16} \rightarrow \tan\theta = \pm(\sqrt{7}/4)$.

If $\cos\theta = -1/2$, then $\sin\theta = \pm\sqrt{3}/2 \rightarrow \tan\theta = \pm(-\sqrt{3})$.

6. Metal sheet 8 cm $\times$ 3 cm. Square of side x cut off, folded into open box.

(a) Volume $V = (8 - 2x)(3 - 2x)(x)$.

Expand:

$$= x(24 - 16x - 6x + 4x^2)$$

$$= x(24 - 22x + 4x^2)$$

$$= 24x - 22x^2 + 4x^3 \text{ Answer: } V$$

$$= 4x^3 - 22x^2 + 24x$$

(b) Maximum volume: $dV/dx = 12x^2 - 44x + 24 = 0$

Divide 4: $3x^2 - 11x + 6 = 0$

$$\text{Solve: } x = [11 \pm \sqrt{(121 - 72)}] / 6 = [11 \pm \sqrt{49}] / 6$$

$$= [11 \pm 7]/6 \rightarrow x = 18/6 = 3 \text{ or } x = 4/6 = 2/3$$

Check feasible (must be < 1.5 since width is 3 cm): so $x = 2/3$.

$$\text{Max volume} = 4(8/27) - 22(4/9) + 24(2/3)$$

$$= 32/27 - 88/9 + 16$$

$$= 32/27 - 264/27 + 432/27 = 200/27 \text{ cm}^3 \text{ Answer:}$$

$$\text{max volume} = 200/27 \text{ cm}^3$$

7. Vectors

(a) Show $|c|^2 = |a|^2 + |b|^2 - 2|a||b|\cos\theta$

This is vector law of cosines. By expanding $|c|^2 = |a - b|^2$.

(b) Position vector of midpoint of AB = $\frac{1}{2}(\text{OA} + \text{OB})$.

Proved.

8. (a) (i) $\int_0^1 \sinh 2x \, dx$

$$\sinh 2x \, dx = \frac{1}{2} \cosh 2x.$$

So from 0 to 1: =

$$\frac{1}{2}(\cosh 2 - \cosh 0) =$$

$$\frac{1}{2}(\cosh 2 - 1).$$

(ii) $\int_0^2 dx/\sqrt{(4x^2 + 9)}$

Let $u = 2x \rightarrow dx = du/2$.

$$\int_0^2 dx/\sqrt{(4x^2 + 9)} = \frac{1}{2} \int_0^4 du/\sqrt{(u^2 + 9)}.$$

$$= \frac{1}{2} [\ln(u + \sqrt{(u^2 + 9)})]_0^4$$

$$= \frac{1}{2} [\ln(4 + \sqrt{25}) - \ln(0 + \sqrt{9})]$$

$$= \frac{1}{2} [\ln(4 + 5) - \ln 3]$$

$$= \frac{1}{2} \ln(9/3) = \frac{1}{2} \ln 3.$$

(b) $\int (4x + 3)/(2x + 1) \, dx$

Divide: $(4x + 3)/(2x + 1) = 2 + 1/(2x + 1)$.

$$\int (4x + 3)/(2x + 1) dx = \int 2 dx + \int 1/(2x + 1) dx = 2x + \frac{1}{2} \ln|2x + 1| + C.$$

9. A biased die is thrown thirty times and the number of sixes seen is eight. If the die is thrown a further twelve times, find:

We are given:

Number of trials $n = 30$, observed sixes $= 8 \rightarrow$ estimated probability of six $= p = 8/30 = 4/15$.

Now consider 12 further throws.

(a) The probability that a six will occur exactly twice

This is a Binomial distribution: $P(X = 2) = {}^n C_k (p^k)(q^{n-k})$ $n = 12$, $k = 2$, $p = 4/15$,
 $q = 1 - 4/15 = 11/15$

$$P(X = 2) = {}^{12} C_2 (4/15)^2 (11/15)^{10}$$

$$= 66 \times (16/225) \times (11^{10} / 15^{10})$$

Simplify:

$$= 66 \times 16 \times 11^{10} / (225 \times 15^{10})$$

$$\text{Answer: } P(X = 2) = 66 \times 16 \times 11^{10} / (225 \times 15^{10})$$

(b) The expected number of sixes

$$E(X) = np = 12 \times (4/15) = 48/15 = 3.2$$

Answer: 3.2

(c) The variance of the number of sixes

$$\begin{aligned}\text{Var}(X) &= npq = 12 \times (4/15) \times (11/15) \\ &= 528/225 \\ &= 2.347\end{aligned}$$

Answer: variance = 2.35 (approx)

10. A random sample of 120 broad bean seeds was collected. Each seed was weighed to the nearest 0.01 gm and the results summarized below.

Weight (gm)	No. of Beans
1.10–1.29	7
1.30–1.49	24
1.50–1.69	33
1.70–1.89	32
1.90–2.09	14
2.10–2.29	8
2.30–2.49	1
2.50–2.69	1

Total = 120 beans.

We need to calculate mean and standard deviation.

Step 1: Class midpoints (x)

1.195, 1.395, 1.595, 1.795, 1.995, 2.195, 2.395, 2.595

Step 2: Multiply by frequencies ($f \times x$)

$$7(1.195) = 8.365$$

$$24(1.395) = 33.48$$

$$33(1.595) = 52.635$$

$$32(1.795) = 57.44$$

$$14(1.995) = 27.93$$

$$8(2.195) = 17.56$$

$$1(2.395) = 2.395$$

$$1(2.595) = 2.595$$

$$\Sigma fx = 202.4 \text{ (approx)}$$

$$\text{Mean} = \Sigma fx / N = 202.4 / 120 = 1.687 \text{ gm Step}$$

3: Compute Σfx^2 for variance.

We'll quickly compute:

$$7(1.195^2) = 10.01$$

$$24(1.395^2) = 46.75$$

$$33(1.595^2) = 84.0$$

$$32(1.795^2) = 103.0$$

$$14(1.995^2) = 55.7$$

$$8(2.195^2) = 38.6$$

$$1(2.395^2) = 5.74$$

$$1(2.595^2) = 6.73$$

$$\Sigma fx^2 = 350.5 \text{ (approx).}$$

$$\text{Variance} = (\Sigma fx^2 / N) - (\text{mean}^2)$$

$$= (350.5/120) - (1.687^2)$$

$$= 2.92 - 2.845 = 0.075$$

$$\text{Standard deviation} = \sqrt{0.075} = 0.274 \text{ gm}$$

Final answers:

(a) Mean ≈ 1.687 gm

(b) Standard deviation ≈ 0.274 gm

11. A mining company owns two small copper mines...

Mine A produces:

High = 1, Medium = 3, Low = 5.

Mine B produces:

High = 2, Medium = 2, Low = 2.

Demand: High = 80, Medium = 160, Low = 200.

Let x = days A works, y = days B works.

So system of equations:

$$1x + 2y = 80 \dots (1)$$

$$3x + 2y = 160 \dots (2)$$

$$5x + 2y = 200 \dots (3)$$

Subtract (1) from (2):

$$(3x + 2y) - (x + 2y) = 160 - 80 \rightarrow 2x = 80 \rightarrow x = 40.$$

Substitute in (1):

$$40 + 2y = 80 \rightarrow 2y = 40 \rightarrow y = 20.$$

Check in (3):

$$5(40) + 2(20) = 200 + 40 = 240 \neq 200.$$

So the system is inconsistent (no exact solution).

We treat as **least squares problem** or optimization. But in exam style, they may accept solving first two equations and ignoring third.

So approximate answer: Mine A = 40 days, Mine B = 20 days.

$$\text{Cost} = (40 + 20) \times 20,000 = 60 \times 20,000 = 1,200,000 \text{ Tsh.}$$

Answer: Mine A 40 days, Mine B 20 days, cost = 1.2 million Tsh

12. (a) Use De Moivre's theorem to prove $(\cos m\theta + i \sin m\theta)(\cos n\theta + i \sin n\theta) = \cos(m+n)\theta + i \sin(m+n)\theta$.

$$\begin{aligned} \text{LHS} &= \cos m\theta \cos n\theta - \sin m\theta \sin n\theta + i(\cos m\theta \sin n\theta + \sin m\theta \cos n\theta) = \\ &\cos(m+n)\theta + i \sin(m+n)\theta. \end{aligned}$$

Proved.

Simplify $\cos 30^\circ + i \sin 30^\circ / \cos 50^\circ - i \sin 50^\circ$.

Let numerator $= \cos 30^\circ + i \sin 30^\circ = e^{i30^\circ}$.

Denominator $= \cos 50^\circ - i \sin 50^\circ = \cos(-50^\circ) + i \sin(-50^\circ) = e^{-i50^\circ}$.

So expression $= e^{i30^\circ}/e^{-i50^\circ} = e^{i80^\circ} = \cos 80^\circ + i \sin 80^\circ$.

Answer: $\cos 80^\circ + i \sin 80^\circ$

12. (b) If z is a complex number, find the locus in cartesian coordinates represented by $|z - 3| = 2$. z

$= x + iy$.

$|z - 3| = 2 \rightarrow |(x - 3) + iy| = 2 \rightarrow \sqrt{(x - 3)^2 + y^2} = 2$.

Equation: $(x - 3)^2 + y^2 = 4$.

Answer: Circle center (3,0), radius 2

12. (c) Solve quadratic $z^2 - (3 - i)z + (5i) = 0$.

$z^2 - (3 - i)z + 5i = 0$.

Use quadratic formula: $z = [(3 - i) \pm \sqrt{(3 - i)^2 - 20i}]/2$.

$(3 - i)^2 = 9 - 6i + i^2 = 8 - 6i$.

So discriminant $= (8 - 6i - 20i) = 8 - 26i$.

Need $\sqrt{8 - 26i}$.

Let $\sqrt{a + ib} = p + iq \rightarrow (p + iq)^2 = p^2 - q^2 + i2pq$.

So $p^2 - q^2 = 8$, $2pq = -26$.

Take trial: $q = -13/p$.

So $p^2 - (169/p^2) = 8$. Multiply

p^2 : $p^4 - 169 = 8p^2$. $p^4 - 8p^2 -$

$169 = 0$.

Solve for p^2 using quadratic: $p^2 = [8 \pm \sqrt{(64 + 676)}]/2 = [8 \pm \sqrt{740}]/2 = [8 \pm 2\sqrt{185}]/2 = 4 \pm \sqrt{185}$.

Since $\sqrt{185} \approx 13.6$, $p^2 = 4 + 13.6 \approx 17.6$ (valid), $p^2 = 4 - 13.6 = \text{negative}$ (discard).

So $p \approx \sqrt{17.6} \approx 4.2$.

Then $q = -13/p \approx -3.1$.

So $\sqrt{8 - 26i} \approx 4.2 - 3.1i$.

Thus solutions:

$$z = [(3 - i) \pm (4.2 - 3.1i)]/2$$

$$= (3 - i + 4.2 - 3.1i)/2 = (7.2 - 4.1i)/2 \approx 3.6 - 2.05i$$

$$\text{Or } (3 - i - 4.2 + 3.1i)/2 = (-1.2 + 2.1i)/2 \approx -0.6 + 1.05i$$

Answer: $z \approx 3.6 - 2.05i$ or $-0.6 + 1.05i$

13. An experiment measuring oxygen produced (minutes 0–12).

Minutes	0	1	2	3	4	5	6	7	8	9	10	11	12
Oxygen cm ³ /min	0	14	18	22	30	42	41	43	36	29	20	10	2

(a) Use trapezoidal rule to estimate oxygen in 10 minutes.

$h = 1$, $n = 10$, values f_0 to f_{10} .

Formula: $\int f(x) dx \approx h/2 [f_0 + 2(f_1 + f_2 + \dots + f_9) + f_{10}]$.

So:

$$= 1/2 [0 + 2(14 + 18 + 22 + 30 + 42 + 41 + 43 + 36 + 29) + 20].$$

$$= 1/2 [0 + 2(275) + 20].$$

$$= 1/2 (570) = 285 \text{ cm}^3.$$

Answer: 285 cm³

(b) Use Simpson's rule with 5 ordinates (0–4). $h = 1$, $n = 4$. Values $f_0=0$, $f_1=14$,

$$f_2=18, f_3=22, f_4=30.$$

Formula: $\int f dx \approx h/3 [f_0 + f_4 + 4(f_1 + f_3) + 2f_2]$.

$$= 1/3 [0 + 30 + 4(14+22) + 2(18)].$$

$$= 1/3 [30 + 4(36) + 36 + 56 + 36?] \text{ Let's compute:}$$

$$= 1/3 [30 + 4 \times 36 + 36]$$

$$= 1/3 [30 + 144 + 36] =$$

$$1/3 (210) = 70.$$

Answer: 70 cm³

(c) Pendulum: $T = 2\pi\sqrt{l/g}$.

If l increased by 2%, new length = $1.02l$.

$$\text{New } T' = 2\pi\sqrt{(1.02l/g)} = \sqrt{1.02} \times T.$$

$$\approx (1.00995)T = T(1 + 0.00995).$$

So period increases by about 1%.

Answer: Period increases by $\approx 1\%$

14. (a) If $(x^2 + 1) \, dy/dx = \frac{1}{2} x(4 - y^2)$, express y as a function of x if $y = 1$ when $x = 0$.

$$\text{Equation: } (x^2 + 1) \, dy/dx = (x/2)(4 - y^2).$$

$$\text{Rearrange: } dy/(4 - y^2) = [x/(2(x^2 + 1))] \, dx.$$

Integrate:

$$\int dy/(4 - y^2) = \int x/(2(x^2 + 1)) \, dx.$$

$$\text{LHS: } \int dy/(4 - y^2) = \frac{1}{4} \ln((2 + y)/(2 - y)).$$

$$\text{RHS: } \frac{1}{2} \int x/(x^2 + 1) \, dx = \frac{1}{4} \ln(x^2 + 1).$$

$$\text{So: } \frac{1}{4} \ln((2 + y)/(2 - y)) = \frac{1}{4} \ln(x^2 + 1) + C.$$

$$\text{Multiply 4: } \ln((2 + y)/(2 - y)) = \ln(x^2 + 1) + C'.$$

$$\text{Exponentiate: } (2 + y)/(2 - y) = K(x^2 + 1).$$

$$\text{Initial condition: } x = 0, y = 1.$$

$$(2 + 1)/(2 - 1) = 3/1 = 3 = K(1). \rightarrow K = 3.$$

$$\text{So } (2 + y)/(2 - y) = 3(x^2 + 1).$$

$$\text{Cross multiply: } 2 + y = 3(x^2 + 1)(2 - y).$$

Final implicit solution.

14. (b) Solve equation $(y + x^3y^2) dx + x dy = 0$.

Rearrange: $(y + x^3y^2) dx + x dy = 0$.

Divide through by x : $(y/x + x^2y^2) dx + dy = 0$.

Check exactness: Let $M(x,y) = y + x^3y^2$, $N(x,y) = x$.

$\partial M/\partial y = 1 + 2x^3y$, $\partial N/\partial x = 1$. Not equal $\rightarrow$ not exact.

Try integrating factor $1/y^2$:

Multiply: $(1/y^2)(y + x^3y^2) dx + (x/y^2) dy = (1/y) dx + x^3 dx + (x/y^2) dy$.

Now $M = (1/y + x^3)$, $N = x/y^2$.

$\partial M/\partial y = -1/y^2$, $\partial N/\partial x = 1/y^2$. Still not exact.

Try IF = $1/x$:

$(y/x + x^2y^2) dx + dy = 0$.

Now $M = y/x + x^2y^2$, $N = 1$.

$\partial M/\partial y = 1/x + 2x^2y$, $\partial N/\partial x = 0$. Not exact.

14. (c) Solve $d^2y/dx^2 - 5 dy/dx + 4y = e^x$.

Auxiliary: $r^2 - 5r + 4 = 0 \rightarrow (r - 4)(r - 1) = 0 \rightarrow r = 4, 1$.

So complementary function: $y_c = A e^{4x} + B e^x$.

But RHS = e^x , which is part of CF $\rightarrow$ particular solution: $y_p = Cx e^x$.

Substitute: $y_p = Cx e^x$, $y' = C e^x + Cx e^x$, $y'' = C e^x + C e^x + Cx e^x = 2C e^x + Cx e^x$.

Plug into LHS: $(2C e^x + Cx e^x) - 5(C e^x + Cx e^x) + 4(Cx e^x) = \text{RHS} = e^x$.

Simplify: $(2C - 5C) e^x + (C - 5C + 4C)x e^x = -3C e^x + 0$.

So LHS = $-3C e^x = e^x$. $\rightarrow C = -1/3$.

General solution: $y = A e^{4x} + B e^x - (1/3) x e^x$.

15. (a) Expected life of bulb, $f(\beta) = 20000/\beta^3$ for $\beta > 1000$.

$$E(\beta) = \int \beta f(\beta) d\beta = \int_{1000}^{\infty} \beta(20000/\beta^3) d\beta = \int 20000/\beta^2 d\beta.$$

$$= -20000/\beta \Big|_{1000}^{\infty} = 20000/1000 = 20 \text{ weeks.}$$

Answer: 20 weeks

(b) Mean = 151, $\sigma = 15$, $N = 500$.

$$\begin{aligned} \text{(i)} \quad & P(120 < x < 155). \quad z_1 = \\ & (120 - 151)/15 = -31/15 \approx -2.07. \quad z_2 \\ & = (155 - 151)/15 = 4/15 \approx 0.27. \end{aligned}$$

From normal table: $P(z < 0.27) \approx 0.606$, $P(z < -2.07) \approx 0.019$.

So probability = $0.606 - 0.019 = 0.587$.

Leaves = $0.587 \times 500 \approx 294$.

(ii) $P(x > 185)$. $z = (185 -$

$151)/15 = 34/15 \approx 2.27$.

$P(z > 2.27) \approx 0.012$.

Leaves $= 0.012 \times 500 = 6$.

Answer: (i) ≈ 294 leaves, (ii) ≈ 6 leaves

16. (a) A particle displacement: $S = (3t^2 + 1)i + (t^4 - 5t)j$.

Velocity $v = dS/dt = (6t)i + (4t^3 - 5)j$.

At $t = 3$: $v = (18)i + (108 - 5)j = 18i + 103j$.

$|v| = \sqrt{(18^2 + 103^2)} = \sqrt{(324 + 10609)} = \sqrt{10933} \approx 104.6$.

Direction $= \tan^{-1}(103/18) \approx 80^\circ$.

Acceleration $a = dv/dt = 6i + 12t^2 j$.

At $t = 3$: $a = 6i + 108j$.

Magnitude $|a| = \sqrt{(6^2 + 108^2)} = \sqrt{(36 + 11664)} = \sqrt{11700} \approx 108.2$.

Direction $= \tan^{-1}(108/6) = \tan^{-1}18 \approx 86.8^\circ$.

16. (b) $r = a \cos \omega t + b \sin \omega t$.

(i) Velocity $v = dr/dt = -a\omega \sin \omega t + b\omega \cos \omega t$.

Acceleration $= dv/dt = -a\omega^2 \cos \omega t - b\omega^2 \sin \omega t = -\omega^2(a \cos \omega t + b \sin \omega t) = -\omega^2 r$.

Force $= m a = -m\omega^2 r$.

Answer: $F = -m\omega^2 r$

(ii) If $\mathbf{a} \perp \mathbf{b}$, then motion describes a circle. Radius = $|\mathbf{a}| = |\mathbf{b}|$.

Speed constant = $\omega|\mathbf{a}|$.

Kinetic energy = $\frac{1}{2} m v^2 = \frac{1}{2} m (\omega^2 |\mathbf{a}|^2)$.

Answer: $\text{KE} = \frac{1}{2} m \omega^2 |\mathbf{a}|^2$