

THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL
ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

142/2

ADVANCED MATHEMATICS 2
(For Both School and Private Candidates)

Time: 3 Hours

Wednesday, March 16, 2005 p.m.

Instructions

1. This paper consists of sections A and B.
2. Answer *all* questions in section A and *four (4)* questions from section B.
3. All necessary steps in answering each question must be shown clearly.
4. Mathematical tables, mathematical formulae, slide rules and non-programmable pocket calculators may be used.
5. Cellular phones are *not* allowed in the examination room.
6. Write your *Examination Number* on every page of your answer booklet(s).

SECTION A (60 marks)

Answer all questions in this section showing all necessary steps and answers.

1. (a) Use logarithms to evaluate:

$$\sqrt[3]{e^{2\sqrt{\ln 3 - \cos 300^\circ}}}$$

(3 marks)

- (b) Using a non-programmable scientific calculator, evaluate:

$$\sqrt{\frac{e^{\log 3} + \sqrt{\log \sqrt{5}}}{\ln 23}}$$

Correct to 6 decimal points.

(3 marks)

2. (a) Let $p \equiv$ He goes abroad.

$q \equiv$ He has a passport.

Write down verbal sentences representing the statements:

(i) $p \leftrightarrow \sim q$

(ii) $\sim p \leftrightarrow q$

(2 marks)

- (b) Construct a truth table for the proposition:

$$[(\sim p \wedge q) \wedge \sim q] \leftrightarrow [q \rightarrow \sim(\sim p \wedge q)].$$

(4 marks)

3. (a) Find an equation of a circle through the points $A(3, -5)$ and $B(2, 6)$ and the line segment $\overline{AB}$ as its diameter.

(2 marks)

- (b) Compute the distance of the tangent line from the point $P(9, 8)$ to the circle which passes through the points $A(5, 7)$, $B(-2, 6)$ and $C(6, 0)$.

(4 marks)

4. (a) Show, by synthetic division that $x = -2$ is a root of a polynomial

$$P(x) = 3x^4 - 8x^3 + 31x^2 + 72x - 92.$$

(2 marks)

- (b) If m and n are the roots of the equation $3x^2 - 7x + 8 = 0$, form quadratic equations whose roots are:

(i) m^2n and n^2m

(ii) m^2 and n^2 .

(4 marks)

5. (a) For small angles θ , the approximate values of $\sin \theta$, $\cos \theta$ and $\tan \theta$ are θ ,

$1 - \frac{1}{2}\theta^2$ and θ respectively. Obtain an expression involving θ that

$$\frac{7 \tan \theta - 20 \cos \theta + 21}{1 + \sin 2\theta} \text{ approximates to, for small values of } \theta.$$

(3 marks)

- (b) Calculate the size of angle A in a triangle for which: $a = 10$, $b = 12$ and $c = 9$.

(3 marks)

6. (a) Show that the shortest distance of the tangent line from the point (10, 10) to the curve $4x^2 + 9y^2 = 25$ at (2, 1) is $\sqrt{145}$ (3 marks)
- (b) Show that the equation: $16x^2 + 25y^2 - 64x + 150y - 111 = 0$ is an equation of an ellipse. (3 marks)
7. (a) By using logarithmic form of hyperbolic functions, evaluate $\sinh^{-1}(3)$. (2 marks)
- (b) Show that a condition for the equation: $a \cosh x + \sinh x + b = 0$, where 'a' and 'b' are real constants, to have real roots is $a^2 - b^2 \leq 1$. (4 marks)
8. (a) The probability that a person supports a certain political party is 0.6. Find the probability that in a randomly selected sample of 8 voters there are exactly 3 persons who support a certain political party. (2½ marks)
- (b) If the probability that it is a fine day is 0.4, find the expected number of fine days in a week, and the standard deviation. (3½ marks)
9. (a) Events A and B are such that $P(A) = \frac{1}{3}$ and $P(A \cap B) = \frac{1}{12}$. If A and B are independent events, find (i) $P(B)$ (ii) $P(A \cup B)$ (3½ marks)
- (b) When a die is thrown, an odd number occurs. What is the probability that the number is prime? (2½ marks)
10. (a) Solve for z and w in the following system of simultaneous equations.

$$\begin{cases} iz - w = 2i \\ iz + iw = 1 \end{cases}$$
 (3 marks)
- (b) Describe the locus of a complex variable z such that $|z - 2 + 3i| \geq 4$ (3 marks)

SECTION B (40 marks)

Answer four (4) questions from this section.

11. (a) Find the shortest distance between the two lines defined by the following equations:
 $\underline{r} = (1-t)\underline{i} + (t-2)\underline{j} + (3-2t)\underline{k}$ and
 $\underline{r} = (s+1)\underline{i} + (2s-1)\underline{j} - (2s+1)\underline{k}$ (5 marks)
- (b) Find the cartesian equation of a plane through the points A (-1, 1, 1) B (2, 1, 0) and C (-2, 0, 3). (5 marks)

12. Find A^{-1} if $A = \begin{pmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{pmatrix}$, hence use the result to solve the following system of equations:

$$\begin{cases} x + 2y - 3z = -4 \\ 2x + 3y + 2z = 2 \\ 3x - 3y - 4z = 11 \end{cases}$$

(10 marks)

13. (a) (i) Change the equation $(x^2 + y^2)^2 = x^2 - y^2$ in polar form.
(ii) Express $r = 4 \sin \theta \cos \theta$ in cartesian form. (5 marks)

- (b) Show that $y = mx + c$ is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
when $c^2 = a^2 m^2 - b^2$ (5 marks)

14. (a) Evaluate $\int_0^2 \frac{dx}{\sqrt{4x^2 + 8x + 13}}$ (5 marks)

- (b) Show that $\int_1^8 \frac{dx}{\sqrt{x^2 - 2x + 2}} = \int_1^2 \frac{3dx}{\sqrt{x^2 - 2x + 2}}$ (5 marks)

15. (a) Below is the truth table containing truth values of two propositions labelled (m) and (n).

P	q	(m)	(n)
T	T	F	F
T	F	T	F
F	T	T	F
F	F	F	T

- (i) Using basic conjunctions, find the propositions (m) and (n), expressing them in terms of connectives $\sim$ and $\rightarrow$ only.

- (ii) Hence using a truth table show that (m) is a tautology and (n) is not.

(5½ marks)

- (b) Check the validity of the argument: "If I like logic, I will study arguments. I will study arguments if and only if I have logical mind. I don't like logic. Therefore I will not study arguments."

(4½ marks)

16. (a) Evaluate the following integral:

$$\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx$$

(2½ marks)

- (b) Use Taylor's theorem to obtain a series expansion for $\cos\left(x + \frac{\pi}{3}\right)$ stating terms up to and including that in x^3 .

(3½ marks)

- (c) Find the minimum value of $3\cosh x + 2\sinh x$.

(4 marks)