

**THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL
ADVANCED CERTIFICATE OF SECONDARY EDUCATION
EXAMINATION**

142/2

ADVANCED MATHEMATICS 2
(For Both School and Private Candidates)

Time: 3 Hours

Thursday, 08th May 2014 a.m.

Instructions

1. This paper consists of sections A and B with a total of **eight (8)** questions.
2. Answer **all** questions in section A and **two (2)** questions from section B.
3. All work done in answering each question must be shown clearly.
4. Mathematical tables and non-programmable calculators may be used.
5. Cellular phones are **not** allowed in the examination room.
6. Write your **Examination Number** on every page of your answer booklet(s).

SECTION A (60 marks)

Answer **all** questions in this section.

1. (a) Obtain the five roots of the equation $z^5 = 32$.
 - (b) If $Q(x, y)$ is a point on the Argand diagram corresponding to $z = x + iy$ and $|z + 2| = 4|z - 2 + 3i|$, find the Cartesian equation of the locus of Q .
 - (c) Prove that $\tan 5\theta = \frac{\tan^5\theta - 10\tan^3\theta + 5\tan\theta}{1 - 10\tan^2\theta + 5\tan^4\theta}$.
 - (d) If $Z = x + iy$ and $\bar{Z}$ is a conjugate of Z , find the values of x and y such that $\left(\frac{1}{Z}\right)^{-1} + \left(\frac{2}{\bar{Z}}\right)^{-1} = 1 + i$.
- (15 marks)

2. (a) Show whether the following statement is valid or not:
If I study hard, then I will not fail Geography. If I do not fail to manage my time then I will study hard. But I failed Geography. Therefore I failed to manage my time.
- (b) Let p be 'He is tall' and q be 'He is handsome' write each of the following statements in symbolic form using p and q :
 - (i) He is tall and handsome.
 - (ii) He is neither tall nor handsome.
- (c) (i) Simplify the compound statement $q \vee (p \wedge q') \vee (r \wedge p')$ using the algebraic laws for logical expressions and then draw the corresponding network.
- (ii) Construct the compound sentences for S_1 and S_2 having the following truth table and simplify S_1 using algebraic laws for logical expressions.

p	q	r	S_1	S_2
T	T	T	F	T
T	T	F	F	F
T	F	T	T	F
T	F	F	F	F
F	T	T	F	T
F	T	F	F	F
F	F	T	T	F
F	F	F	F	F

- (d) Use a truth table to show that the statement $\sim P \wedge [Q \wedge (\sim Q \vee P)]$ is a self-contradiction.

(15 marks)

3. (a) Forces of magnitude 5 and 7 units acting in the direction of $2\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$ and $3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ respectively act on a particle which is displaced from the point $(2, 2, -1)$ to $(4, 3, 1)$. Find the work done by the forces.

(b) A particle with 200g of mass is moving along a curve with the velocity $\frac{\mathbf{i}}{4t} - \frac{7\mathbf{j}}{t} + \frac{\mathbf{k}}{1-2t}$.

(i) Find the force applied to the particle at any time t .

(ii) Find the position vector at time t where the particle is heading.

(c) Find the area of a parallelogram having the diagonals $\mathbf{A} = 3\mathbf{i} + \mathbf{j} - 2\mathbf{k}$ and $\mathbf{B} = \mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$ (leave your answer in surd form).

(d) Use the definition of dot product to prove the law of cosine for a triangle ABC.

(15 marks)

4. (a) (i) Find the adjoint of $\begin{pmatrix} 3 & -2 & -2 \\ 2 & 3 & -1 \\ 1 & -1 & 3 \end{pmatrix}$.

(ii) Use the adjoint obtained in 4 (a) (i) to solve the system of equations

$$3x - 2y - 2z = 1$$

$$2x + 3y - z = 13$$

$$x - y + 3z = -8$$

(b) If x is so small that x^3 and higher powers may be neglected, obtain a quadratic approximation of $f(x) = \frac{1 + e^x}{1 + \ln(1+x)}$.

(c) Express $\frac{2x+5}{x^2+3x+2}$ in partial fractions and hence state the first four terms in the series expansion of $\frac{2x+5}{x^2+3x+2}$.

(d) Write the following series in the sigma notation:

(i) $\frac{2}{7} - \frac{4}{7} + \frac{8}{7} - \frac{16}{7} + \dots + \frac{128}{7}$,

(ii) $\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \dots + \frac{1}{42}$.

(15 marks)

SECTION B (40 marks)

Answer any **two (2)** questions from this section. Extra questions will not be marked.

5. (a) Show that $2 \tan^{-1} 2 + \tan^{-1} 3 = \pi + \tan^{-1} \frac{1}{3}$.
- (b) Find all angles which satisfy the equation $10 \sec^2 \theta - 3 = 17 \tan \theta$.
- (c) (i) Show that $\cos 3A = 4\cos^3 A - 3\cos A$.
- (ii) By using the results in part (c)(i) express $\cos^2 A (16\cos^4 A - 24\cos^2 A + 9)$ as a single term of cosine.
- (iii) If α , β and γ are the angles of a triangle, prove that
- $$\cos \alpha + \cos \beta + \cos \gamma - 1 = 4 \sin \frac{\alpha}{2} \sin \frac{\beta}{2} \sin \frac{\gamma}{2}.$$
- (d) If $x \sin^3 \theta + y \cos^3 \theta = \sin \theta \cos \theta$ and $x \sin \theta - y \cos \theta = 0$, prove that $x^2 + y^2 = 1$.
(20 marks)

6. (a) The random variable X has the following probability table:

Value of X	0	1	2
Probability of the value of X	k	$2k$	$3k$

- (i) Determine the value of the constant k .
- (ii) Find $E(X)$.
- (b) If a random variable X has probability density function
- $$f(x) = \begin{cases} kx, & 0 \leq x \leq 2 \\ k(4-x), & 2 \leq x \leq 4 \\ 0, & \text{otherwise} \end{cases}$$
- (i) Find the value of the constant k .
- (ii) Evaluate $P\left(\frac{1}{2} \leq X \leq \frac{5}{2}\right)$.
- (c) In a class of 20 students, each has 45% chance to pass CSEE 2014. Find the probability that;
- (i) Exactly 15 students will pass.
- (ii) At least two students will pass.
- (iii) More than 17 students will pass.
(Write your answers to six decimal places).
- (d) Ten percent of the tools produced in a certain manufacturing process turn out to be defective. Find the probability that in a sample of 20 tools chosen at random exactly 5 will be defective, by using
- (i) The binomial distribution.
- (ii) Poisson approximation to the binomial distribution.
(Write your answers to four decimal places).
(20 marks)

7. (a) Obtain the first order differential equation of $y = cx^2 + c^2$.
- (b) Find the general solution of the differential equation $\frac{dy}{dx} = \frac{2x + y - 2}{2x + y + 1}$.
- (c) Find a curve on the x-y plane that passes through the origin and for which the tangent at (x, y) has slope $x^2 + y$.
- (d) Solve the differential equation $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} - 3x = 2\cos t - 4\sin t$ given the initial conditions $x = 2$ and $\frac{dx}{dt} = -3$ at $t = 0$.
- (e) Determine whether or not the following functions are solutions of the differential equation $\frac{d^2y}{dx^2} - y = 0$. (i) $y = 2\sin x$ (ii) $y = e^{-2x}$ and (iii) $y = \frac{4}{e^x}$.

(20 marks)

8. (a) (i) Find the equation of the normal to the parabola $\frac{y^2}{16x} = p$ at the parametric coordinates $\left(\frac{pt^4}{4}, 2pt^2\right)$ where p is constant.
- (ii) Find the equation of the tangent to the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ at the parametric coordinates $(3\cos\theta, 2\sin\theta)$.
- (b) (i) Derive the equations of asymptotes of a hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.
- (ii) Find the eccentricity and foci of the curve $\frac{(x-2)^2}{4} - \frac{(y-3)^2}{9} = 1$.
- (c) Prove that the equation $r = \frac{4}{1 + \cos\theta}$ represents a translated parabola.
- (d) Given the equation $y + \frac{x^2 - 10x + 25}{12} - 4 = 0$:
- (i) Write the equation in the standard form of the parabola.
- (ii) Find the line of symmetry of the parabola in (d) (i).
- (iii) Find the focus of the parabola in (d) (i).

(20 marks)