

**THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL OF TANZANIA
ADVANCED CERTIFICATE OF SECONDARY EDUCATION
EXAMINATION**

142/2

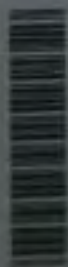
ADVANCED MATHEMATICS 2
(For Both School and Private Candidates)

Time: 3 Hours

Thursday, 05th May 2016 a.m.

Instructions

1. This paper consists of **eight (8)** questions in sections A and B.
2. Answer **all** questions in section A and **two (2)** questions from section B.
3. All work done in answering each question must be shown clearly.
4. Mathematical tables and non-programmable calculators may be used.
5. Cellular phones are **not** allowed in the examination room.
6. Write your **Examination Number** on every page of your answer booklet(s).



SECTION A (60 Marks)

Answer **all** questions in this section.

1. (a) (i) Use the De Moivre's theorem to find the value of $(1+i)^8$.
 (ii) Use the mathematical induction to prove that
 $(r(\cos\theta + i\sin\theta))^n = r^n(\cos n\theta + i\sin n\theta)$
 - (b) (i) If $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{4}$ and $z = x + iy$, find the locus of the point representing z in an argand diagram.
 (ii) Solve the following system of equations where z and w are complex numbers.

$$\begin{cases} iz - w = 2i \\ iz + iw = i \end{cases}$$
 - (c) One of the roots of the equation $z^4 - 6z^2 + 23z^2 - 34z + 26 = 0$ is $1+i$. Find the other roots. **(15 marks)**
2. (a) (i) Draw the simplified electrical circuit for the argument
 $[p \wedge (p \vee q)] \vee [q \wedge \neg (p \wedge q)]$
 (ii) Use the truth table values only to show whether or not $p \leftrightarrow \neg q$ and $\neg (p \wedge q)$ are logically equivalent.
 - (b) (i) Use the truth table to test the validity of the following argument: If I am intelligent, then I will pass this examination. I am intelligent. Therefore I will pass this examination.
 (ii) Write the converse, inverse and contrapositive of the statement 'If Mathematics is interesting then Biology is boring and tough'. **(15 marks)**
3. (a) If the position vector $\vec{OA}$, $\vec{OB}$ and $\vec{OC}$ are defined by $\vec{OA} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$,
 $\vec{OB} = 3\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$ and $\vec{OC} = -\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$;
 (i) Determine the cross product $\vec{AB} \times \vec{BC}$.
 (ii) Find the exact value of the angle between $\vec{AB}$ and $\vec{BC}$.
 - (b) If $\underline{a} = 3\mathbf{i} + 2\mathbf{j} + 9\mathbf{k}$ and $\underline{b} = \mathbf{i} + \lambda\mathbf{j} + 3\mathbf{k}$. Find
 (i) The value of λ so that $\underline{a} + \underline{b}$ is perpendicular to $\underline{a} - \underline{b}$.
 (ii) The projection of $\underline{a}$ onto $\underline{b}$ and leave the answer in surd form.
 - (c) Derive the cosine's rule using the vectors $\underline{u}$ and $\underline{v}$. **(15 marks)**
4. (a) (i) Find the value of a if the 17^{th} and 18^{th} terms of the expansion $(2+a)^{30}$ are equal.

- (ii) The roots of the equation $x^2 + px^2 + qx + 30 = 0$ are in the ratio 2:3:5. Find the value of p and q .
- (b) (i) State the principle of Mathematical Induction as it is used in mathematics.
- (ii) Use the principle of mathematical induction to prove that $\sum_{r=1}^n 3r - 1 = \frac{n}{2}(3n + 1)$
- (15 marks)

SECTION B (40 Marks)

Answer any **two (2)** questions from this section. Extra questions will not be marked.

5. (a) (i) Solve the equation $\tan^{-1}\left(\frac{x-1}{x+2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$ and leave the answer in surd form.
- (ii) Prove the identity $\frac{1 + \sin x}{1 - \sin x} \equiv (\tan x + \sec x)^2$.
- (b) If $2\sin\theta + \cos\theta = 1$, use t-formula to find the value of θ in the interval $0^\circ \leq \theta \leq 180^\circ$.
- (c) (i) Show that $\frac{\cos\theta + \cos 2\theta + \cos 3\theta + \cos 4\theta}{\sin\theta + \sin 2\theta + \sin 3\theta + \sin 4\theta} = \cot\left(\frac{5\theta}{2}\right)$.
- (ii) Verify that $\frac{\sin(A+B+C) + \sin(A-B-C)}{\cos(A+B+C) - \cos(A-B-C)} = \frac{\tan B \tan C - 1}{\tan B + \tan C}$.
- (d) Express $3\sin\theta - 4\cos\theta$ in the form $R\sin(\theta - \alpha)$ giving values of R and α . (20 marks)
6. (a) The probability that a keyboard picked at random from the assembly line in a factory will be defective is 0.01. If a sample of three is to be selected:
- (i) Construct the probability distribution of the defective keyboards.
- (ii) Find the mean and standard deviation (Give your answers correct to 2 decimal places).
- (b) The bag R contains 5 red and 3 green balls and bag P contains 3 red and 5 green balls. If one ball is drawn from bag R and two from bag P, find the probability that out of three balls drawn two are red and one is green.
- (c) The random variable X has a probability distribution $P(x)$ of the following form, where k is a certain number.
- $$P(X) = \begin{cases} k & \text{if } x = 0 \\ 2k & \text{if } x = 1 \\ 3k & \text{if } x = 2 \\ 0 & \text{otherwise} \end{cases}$$

- (i) Determine the value of k .
- (ii) Find $P\{x < 2\}$, $P\{x \leq 2\}$ and $P\{x \geq 2\}$.

- (d) (i) If X is a discrete random variable where $E\{X\}$ is the expected value of X , show that $E\{Ax + b\} = aE\{x\} + b$ where a and b are constants.
- (ii) The modern seeds of a certain crop have the probability of germinating 0.9. If six seeds are sown, what is the probability of at most 5 seeds are germinating?
(20 marks)

7. (a) (i) If $x(1-y)\frac{dy}{dx} + 2y = 0$ and $y = 2$ when $x = e$, show that $x^2ye^{-y} = 2$.
- (ii) Solve the differential equation $(2x - y)\frac{dy}{dx} = 2x - y + 2$ given that $y = 1$ when $x = 2$.

- (b) Form a differential equation whose solution is the function $y = Ae^{2x} + Be^{-2x}$ where A and B are arbitrary constants.

- (c) A tank contains a solution of salt in water. Initially the tank contains 1000 litres of water with 10 kg of salt dissolved in it. The mixture is poured off at a rate of 20 litres per minute, and simultaneously pure water is added at a rate of 20 litres per minute. All the time the tank is stirred to keep the mixture uniform.

- (i) Find the mass of the salt in the tank after 5 minutes.
- (ii) How long the mass of the salt in the tank falls to 5 kg?

- (d) Find the general solution of the differential equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 10e^{-2x}$.

(20 marks)

8. (a) (i) If $9y^2 - 54y - 25x^2 + 200x - 544 = 0$ is the hyperbola equation; find the center, the vertices, the foci and the equation of the asymptotes.

- (ii) Convert $x^2 + y^2 = 4x$ into polar equation.

- (iii) Convert $(1, -1)$ into polar coordinates.

- (b) Find the equation of the tangent and normal at $P(a\cos u, b\sin u)$ to the ellipse $b^2x^2 + a^2y^2 = a^2b^2$.

- (c) (i) Define a conic section.

- (ii) A man running a race-course notes that the sum of the distances from the two flag posts to him is always 10 meters. If the distance between the flag posts is 8 meters, find the equation of the path traced by the man.

- (d) Sketch the graph of $r = 2(1 + \sin t)$.

(20 marks)