

**THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL OF TANZANIA
ADVANCED CERTIFICATE OF SECONDARY EDUCATION
EXAMINATION**

142/2

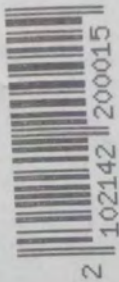
ADVANCED MATHEMATICS 2
(For Both School and Private Candidates)

Year : 2021

Time: 3 Hours

Instructions

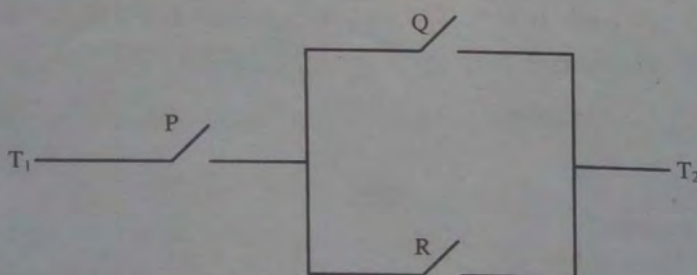
1. This paper consists sections A and B with a total of **eight (8)** questions.
2. Answer **all** questions in section A and **two (2)** questions from section B.
3. Section A carries **sixty (60)** marks and section B carries **forty (40)** marks.
4. All work done in answering each question must be shown clearly.
5. NECTA'S mathematical tables and non-programmable calculators may be used.
6. Cellular phones and any unauthorised materials are **not** allowed in the examination room.
7. Write your **Examination Number** on every page of your answer booklet(s).



SECTION A (60 Marks)

Answer **all** questions in this section.

1. (a) If A and B are such that $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$ and $P(A \cup B) = \frac{1}{2}$, calculate;
 - (i) $P(A \cap B')$.
 - (ii) $P(A/B')$.
 - (b) Two dices are thrown simultaneously.
 - (i) List the sample space for this event.
 - (ii) Find the probability that the sum of the numbers obtained on the dice is neither a multiple of 2 nor a multiple of 3.
 - (c) If X is binomially distributed, the probability that the event will happen exactly x times in n trials is given by the function $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$. Establish the validity of the Poisson approximation to the binomial distribution.
2. (a) The contrapositive of the statement Y is given by $\sim(Q \wedge P) \rightarrow \sim P$. By using the laws of algebra of propositions, show that its inverse is a tautology.
 - (b) Test the validity of the argument whose conclusion is $\sim Q$ and premises are $P \rightarrow (\sim P \rightarrow Q)$, $Q \rightarrow \sim P$ and P .
 - (c) (i) Construct a truth table for the compound statement that corresponds to the following circuit:



- (ii) Draw a simple network diagram for the statement $(P \rightarrow Q) \wedge (P \vee Q)$.
3. (a) If $\underline{a} = \underline{i} - \underline{j} + 2\underline{k}$ and $\underline{b} = \underline{i} + \underline{j}$, find the unit vector orthogonal to both $\underline{a}$ and $\underline{b}$.
 - (b) The position vectors of the points A and B are $2\underline{i} + 3\underline{j} - \underline{k}$ and $-\underline{i} + 5\underline{j} + 7\underline{k}$ respectively. If C divides $\overline{AB}$ internally in the ratio 2:1, find the position vector of point C.
 - (c) Using the cosine rule, show that in the triangle ABC, $\underline{c} = \underline{b} \cos A + \underline{a} \cos B$.

4. (a) If $2x + 10yi - 4y = -12 + 5i$, find the values of x and y .
 (b) Express $(\cos \theta + i \sin \theta)^{-n}$ in the form $a + ib$.
 (c) Given that $z = x + iy$, express the complex number $\frac{z+i}{iz+2}$ in polynomial form and hence find the resulting complex number when $z = 1 + 2i$.

SECTION B (40 Marks)

Answer **two (2)** questions from this section.

5. (a) If A , B and C are angles of a right angled triangle such that $\cos A = \frac{3}{5}$ and $\cos B = \frac{5}{13}$, find the value of $\tan 2A$, $\cos(A+B)$ and $\operatorname{cosec}(A-B)$ in the form $\frac{x}{y}$.
 (b) (i) Show that $\cot\left(x + \frac{\pi}{2}\right) - \tan\left(x - \frac{\pi}{2}\right) = \frac{2 \cos 2x}{\sin 2x}$.
 (ii) Solve the equation $4 \cos 2\theta - 2 \cos \theta + 3 = 0$, for $0^\circ \leq \theta \leq 360^\circ$.
 (c) Express $\cos^4 \theta$ in terms of cosines multiples of θ .
6. (a) By using the first five terms in the expansion of $(1+x)^n$, find the value of $(1.98)^{10}$ correct to three decimal places.
 (b) The polynomial $x^5 + 4x^2 + ax + b$ leaves the remainder of $2x + 3$ when it is divided by $x^2 - 1$. Use the remainder theorem to find the values of a and b .
 (c) The roots of the quadratic equation $x^2 + 2mx + n = 0$ differ by 2. Show that $m^2 = 1 + n$.
 (d) If $A = \begin{pmatrix} 4 & -1 & 1 \\ 0 & 0 & 2 \\ m & -1 & 1 \end{pmatrix}$ is singular, find the value of m .
 (e) Use Cramer's rule to solve the following system of equations:
$$\begin{cases} 5x + 6y + 4z = 5 \\ 7x - 4y - 3z = 8 \\ 2x + 3y + 2z = 2 \end{cases}$$
7. (a) Form a differential equation whose solution is $x = \tan(Ay)$.
 (b) Solve the differential equation $\frac{d^2 \theta}{dt^2} - 4 \frac{d\theta}{dt} + 4\theta = \frac{3}{7}$.
 (c) A biologist is researching the population of a specie. She tries a number of different models for the rate of growth of the population and solves them to compare with

observed data. Her first model is $\frac{dn}{dt} = kn\left(1 - \frac{n}{a}\right)$ where n is the population at time t years, k is a constant and a is the maximum population sustainable by the environment. Given that $k = 0.2$, $a = 100000$ and the initial population is 30000;

- (i) find the general solution of the differential equation.
- (ii) estimate the population after 5 years to 2 significant figures.

8. (a) Express $x^2 + y^2 = 2x + 2y$ in polar form.
- (b) Find the equation of the chord of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ joining the points whose eccentric angles are θ and ϕ .
- (c) Show that $P(a \sec \theta, b \tan \theta)$ lies on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, hence find the equation of the tangent line at point P on the given hyperbola.
- (d) Show whether the equation of a normal to the parabola $y^2 = 4ax$ at point (x_1, y_1) is $(x - x_1)y_1 + 2a(y - y_1) = 0$.
- (e) (i) Change the polar equation $r^2(b^2 \cos^2 \theta + a^2 \sin^2 \theta) = a^2 b^2$ into the Cartesian equation.
- (ii) Draw the graph of $r = 2(1 + \cos \theta)$.