

**THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL OF TANZANIA
ADVANCED CERTIFICATE OF SECONDARY EDUCATION
EXAMINATION**

142/2

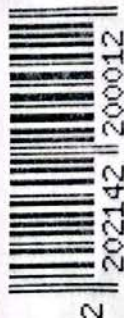
ADVANCED MATHEMATICS 2
(For Both School and Private Candidates)

Time: 3 Hours

Year: 2022

Instructions

1. This paper consists sections A and B with a total of **eight (8)** questions.
2. Answer **all** questions in section A and **two (2)** questions from section B.
3. Section A carries **sixty (60)** marks and section B carries **forty (40)** marks.
4. All work done in answering each question must be shown clearly.
5. NECTA's Mathematical tables and non-programmable calculators may be used.
6. Cellular phones and any unauthorised materials are **not** allowed in the examination room.
7. Write your **Examination Number** on every page of your answer booklet(s).



SECTION A (60 Marks)

Answer **all** questions in this section.

1. (a) The time taken by John to deliver milk to the High Street is normally distributed with mean 12 minutes and standard deviation 2 minutes. If he delivers milk every day, estimate the number of days during the year when he takes longer than 17 minutes. (1 year = 365 days)
- (b) Suppose that a group of people in a village attending hospital has been categorized according to the incidence of two diseases

Sex	Malaria	Typhoid
Male	16	12
Female	12	10

Find the probability that the person chosen is a female given that the person is suffering from malaria.

- (c) In how many ways can a hand of 4 cards be chosen from an ordinary pack of 52 playing cards?
2. (a) Write the converse and inverse of the statement "If you score an A grade in a logic test, then I will buy you a new car" in words and symbolic form.
- (b) Using a truth table, examine whether $[(\sim p) \rightarrow (\sim q)] \wedge (p \rightarrow q)$ is equivalent to $(q \rightarrow p) \wedge (p \rightarrow q)$.
- (c) Use laws of algebra of propositions to simplify $[p \wedge (p \vee q)] \vee [q \wedge (\sim (p \wedge q))]$.
3. (a) Find the work done by force $\underline{F} = i + 2j + k$ moving an object at a distance of 7 m in the direction of the vector $\underline{r} = 3i + 2j + 4k$.
- (b) If P and Q are points $P(3, -4, 6)$ and $Q(1, -3, 8)$ respectively, find a unit vector parallel to the displacement vector $\overrightarrow{PQ}$.
- (c) The position vectors of points A and B are $\underline{a}$ and $\underline{b}$ respectively. If point C divides $\overline{AB}$ internally in the ratio of 2:1, D divides $\overline{AB}$ externally in the ratio of 1:4 and E divides $\overline{CD}$ internally in the ratio of 2:1, find the position vectors of C, D and E in terms of $\underline{a}$ and $\underline{b}$.
4. (a) Express $\sqrt{1+i}$ in polar form.
- (b) Using the results in part (a), show that $\tan \frac{\pi}{8} = \sqrt{2} - 1$.
- (c) If $z_1 = r_1(\cos \theta_1 + i \sin \theta_1)$ and $z_2 = r_2(\cos \theta_2 + i \sin \theta_2)$; Prove that
$$\text{Arg} \left(\frac{z_1}{z_2} \right) = \text{Arg}(z_1) - \text{Arg}(z_2).$$
- (d) The complex numbers $z_1 = \frac{c}{1+i}$ and $z_2 = \frac{d}{1+2i}$ where $c, d \in \mathbb{R}$ are such that $z_1 + z_2 = 1$, find the values of c and d .

SECTION B (40 Marks)

Answer two (2) questions from this section.

5. (a) For all values of α show that $\frac{\sin 3\alpha}{\sin \alpha} - \frac{\cos 3\alpha}{\cos \alpha} = 2$.
- (b) Prove that $\frac{\sin x + \sin 2x + \sin 3x}{\cos x + \cos 2x + \cos 3x} = \tan 2x$.
- (c) Solve for β in the trigonometric equation $\tan^{-1}\left(\frac{\beta-1}{\beta-2}\right) + \tan^{-1}\left(\frac{\beta+1}{\beta+2}\right) = \frac{\pi}{4}$.
- (d) Rewrite $4\cos\theta + 3\sin\theta$ in the form $R\cos(\theta - \alpha)$, hence solve the equation $4\cos\theta + 3\sin\theta = \frac{5\sqrt{2}}{2}$ in the interval $\frac{\pi}{2} \leq \theta \leq 2\pi$.
6. (a) If the coefficients of x and x^2 in the expansion of $\frac{1+px+qx^2}{(1-x)^2}$ are zero, find the numerical values of p and q .
- (b) Use the principle of mathematical induction to prove that for every positive integer, $3^{2n-2} + 2^{6n}$ is divisible by 5.
- (c) Given that $P(x) = 2x^3 + 7x^2 - 5$, use the synthetic method to find the quotient and remainder when $P(x)$ is divided by $x+3$.
- (d) Find the determinant and inverse of the matrix $A = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 5 & 2 \\ 1 & -1 & 1 \end{pmatrix}$, hence solve the simultaneous equations
$$\begin{cases} 2x + y = 4 \\ x + 5y + 2z = 7 \\ x - y + z = 1 \end{cases}$$
7. (a) (i) Determine the most general function $M(x, y)$ such that the differential equation $M(x, y)dx + (2x^2y^3 + x^4y)dy = 0$ is exact.
- (ii) By separating the variables, solve the differential equation $(xy + x)dx - (x^2y^2 + x^2 + y^2 + 1)dy = 0$
- (b) Find the general solution of the differential equation $\cos x \frac{d^2y}{dx^2} - \sin x \frac{dy}{dx} = 0$.
- (c) A liquid of 72°C placed in a room at 25°C has a temperature of 65°C after 5 minutes. Find its temperature after further 10 minute.
- (d) Formulate a differential equation of a circle which passes through the origin and whose centre lies on the y-axis.

8. (a) Find the coordinates of the foci, the vertices, the eccentricity and the length of the latus rectum of the hyperbola $16x^2 - 9y^2 = 576$.
- (b) (i) Determine the equation of the normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point $P(a \cos \theta, b \sin \theta)$.
- (ii) If the normal in part (b) (i) meets the x-axis at A and the y-axis at B, find the area of the triangle AOB where O is the origin.
- (c) Show that the equation of the tangent to the parabola $y^2 = 4ax$ at the point $(at^2, 2at)$ is $x - ty + at^2 = 0$.
- (d) (i) Change the Cartesian equation $(x^2 + y^2)^3 - 2xy(x^2 - y^2)$ into a polar equation.
- (ii) Sketch the graph of $r = 1 - 2 \cos \theta$ from $\theta = 0$ to $\theta = 2\pi$.