

THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATION COUNCIL OF TANZANIA
ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

142/1

ADVANCED MATHEMATICS 1
(For Both Private and School Candidates)

Duration: 3 Hour.

ANSWERS

Year: 2025

Instructions

1. This paper consists of **ten (10)** questions.
2. Answer **all** questions.
3. Write your **Examination Number** on every page of your answer booklet(s).

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1. (a) Use a non-programmable calculator to evaluate the following expressions:

(i) $\frac{\tan 25^\circ 30' \div \sqrt{(0.03e^{-3})}}{\ln 3.2 + 0.006e^{0.33}}$ correctly to six significant figures

(ii) $\sum$ from $n = 1$ to 5 of $(2^n \times n!) \div \ln(0.3n)$, correct to three decimal places.

(i)

Convert $25^\circ 30'$ to decimal degrees:

$$25^\circ 30' = 25.5^\circ$$

$$\tan(25.5^\circ) = 0.4778$$

$$e^{-3} = 0.04979$$

$$0.03 \times e^{-3} = 0.03 \times 0.04979 = 0.0014937$$

$$\sqrt[3]{(0.0014937)} = 0.03866$$

So:

$$\tan(25.5^\circ) \div \sqrt[3]{(0.03e^{-3})} = 0.4778 \div 0.03866 = 12.3627$$

$$\ln(3.2) = 1.1632$$

$$e^{0.33} = 1.391$$

$$0.006 \times 1.391 = 0.008346$$

$$\ln(3.2) + 0.008346 = 1.1715$$

(ii)

We compute the following:

$$\sum \text{from } n = 1 \text{ to } 5 \text{ of } (2^n \times n!) \div \ln(0.3n)$$

When $n = 1$:

$$2^1 \times 1! = 2$$

$$\ln(0.3 \times 1) = \ln(0.3) = -1.2040$$

$$2 \div -1.2040 = -1.6627$$

When $n = 2$:

$$2^2 \times 2! = 4 \times 2 = 8$$

$$\ln(0.3 \times 2) = \ln(0.6) = -0.5108$$

$$8 \div -0.5108 = -15.6582$$

When $n = 3$:

$$2^3 \times 3! = 8 \times 6 = 48$$

$$\ln(0.3 \times 3) = \ln(0.9) = -0.1054$$

$$48 \div -0.1054 = -455.4292$$

When $n = 4$:

$$2^4 \times 4! = 16 \times 24 = 384$$

$$\ln(0.3 \times 4) = \ln(1.2) = 0.182$$

$$384 \div 0.182 = 2109.8901$$

When $n = 5$:

$$2^5 \times 5! = 32 \times 120 = 3840$$

$$\ln(0.3 \times 5) = \ln(1.5) = 0.4055$$

$$3840 \div 0.4055 = 9470.4317$$

Now sum all values:

$$-1.6627 + (-15.6582) + (-455.4292) + 2109.8901 + 9470.4317 = 11107.5717$$

Answer = 11107.572 (to three decimal places)

(b) If $A_n = A_5 + A_{20}e^{-kn}$, where $k = 0.1386$, $A_5 = 20$ and $A_{20} = 40$, find the value of A_{16} .

We calculate:

$$A_{16} = 20 + 40 \times e^{-(0.1386 \times 16)}$$

$$0.1386 \times 16 = 2.2176$$

$$e^{-2.2176} = 0.1086$$

$$40 \times 0.1086 = 4.344$$

$$20 + 4.344 = 24.344$$

Answer: $A_{16} = 24.34$ (to 4 significant figures)

2. (a) Prove that $(1 + \tanh x) \div (1 - \tanh x) = \cosh 2x + \sinh 2x$.

We start with the identity:

$$\tanh x = \sinh x \div \cosh x$$

Now compute the left-hand side:

$$(1 + \tanh x) \div (1 - \tanh x)$$

$$= (1 + \sinh x \div \cosh x) \div (1 - \sinh x \div \cosh x)$$

$$= (\cosh x + \sinh x) \div (\cosh x - \sinh x)$$

Now multiply both numerator and denominator by $(\cosh x + \sinh x)$:

$$= [(\cosh x + \sinh x)^2] \div [\cosh^2 x - \sinh^2 x]$$

Recall that:

$$\cosh^2 x - \sinh^2 x = 1$$

So the denominator becomes 1.

Now expand the numerator:

$$(\cosh x + \sinh x)^2 = \cosh^2 x + 2 \sinh x \cosh x + \sinh^2 x$$

$$= (\cosh^2 x + \sinh^2 x) + 2 \sinh x \cosh x$$

$$= 1 + 2 \sinh x \cosh x$$

But:

$$\cosh 2x = \cosh^2 x + \sinh^2 x = 1 + 2 \sinh^2 x$$

$$\sinh 2x = 2 \sinh x \cosh x$$

So:

$$1 + 2 \sinh x \cosh x = \cosh 2x + \sinh 2x$$

Therefore:

$$(1 + \tanh x) \div (1 - \tanh x) = \cosh 2x + \sinh 2x$$

(b) Differentiate $\ln(\tanh x)$ with respect to x and simplify your answer.

$$\text{Let } y = \ln(\tanh x)$$

$$dy/dx = 1 \div \tanh x \times d/dx(\tanh x)$$

$$d/dx(\tanh x) = \operatorname{sech}^2 x$$

So:

$$dy/dx = \operatorname{sech}^2 x \div \tanh x$$

$$\text{Answer: } dy/dx = \operatorname{sech}^2 x \div \tanh x$$

(c) Find the area enclosed by the curve $y = \cosh x$, the lines $x = \ln 2$, the x -axis and y -axis. Hence find the volume obtained when this area is rotated completely about the x -axis.

$$\text{Area } A = \int \text{from } 0 \text{ to } \ln 2 \text{ of } \cosh x \, dx$$

$$\int \cosh x \, dx = \sinh x$$

So:

$$A = \sinh(\ln 2) - \sinh(0) = \sinh(\ln 2)$$

$$\begin{aligned} \text{Now, } \sinh(\ln 2) &= (e^{(\ln 2)} - e^{(-\ln 2)}) \div 2 \\ &= (2 - 0.5) \div 2 = 1.5 \div 2 = 0.75 \end{aligned}$$

$$\text{Therefore, Area} = 0.75 \text{ units}^2$$

Now for volume when rotated about x -axis:

$$\text{Volume } V = \pi \int \text{from } 0 \text{ to } \ln 2 \text{ of } (\cosh x)^2 \, dx$$

We use identity:

$$\cosh^2 x = (1 + \cosh 2x) \div 2$$

So:

$$\int \cosh^2 x \, dx = \frac{1}{2} \int (1 + \cosh 2x) \, dx$$

$$= \frac{1}{2} [x + \frac{1}{2} \sinh 2x] + C$$

Now apply the limits from 0 to $\ln 2$:

$$x = \ln 2$$

$$\sinh 2x = \sinh(2 \ln 2)$$

$$= (e^{(2 \ln 2)} - e^{(-2 \ln 2)}) \div 2 = (4 - 0.25) \div 2 = 3.75 \div 2 = 1.875$$

Then:

$$V = \pi \times \frac{1}{2} [\ln 2 + \frac{1}{2} \times 1.875]$$

$$= \pi \times \frac{1}{2} [\ln 2 + 0.9375]$$

$$\ln 2 \approx 0.6931$$

$$= \pi \times \frac{1}{2} \times (0.6931 + 0.9375)$$

$$= \pi \times \frac{1}{2} \times 1.6306 = \pi \times 0.8153$$

$$\approx 3.1416 \times 0.8153 = 2.5611$$

Answer:

$$\text{Area} = 0.75$$

$$\text{Volume} \approx 2.5611 \text{ units}^3$$

3. Rehema has 900 tonnes and 600 tonnes of bricks at Mtakuja and Tupendane villages respectively. She has planned to build new houses at sites A, B and C. She expects to use 500 tonnes of bricks at site A, 600 tonnes at site B and 400 tonnes at site C. The transport cost in Tanzanian shillings is proportional to the distance covered in kilometres as shown in the following table:

From/To	A	B	C
Mtakuja	600	300	400
Tupendane	400	200	600

If x and y are the number of bricks to be transported from Mtakuja to sites A and B respectively:

(a) Formulate the inequalities and the objective function to be satisfied by x and y .

Let x be the number of bricks from Mtakuja to site A.

Let y be the number of bricks from Mtakuja to site B.

Total bricks available at Mtakuja is 900 tonnes, so:

$$x + y + z \leq 900, \text{ where } z \text{ is bricks sent from Mtakuja to site C.}$$

But since site C needs 400 tonnes in total and total site needs = $500 + 600 + 400 = 1500$ tonnes, and Mtakuja has 900 tonnes while Tupendane has 600 tonnes, all bricks will be used.

Also, we know:

$$\text{Bricks to site A: } x \text{ (from Mtakuja) } + (500 - x) \text{ (from Tupendane)}$$

Bricks to site B: y (from Mtakuja) + $(600 - y)$ (from Tupendane)

Bricks to site C: 400 (can be from either source)

So, the constraints are:

$x + y \leq 900$ (supply from Mtakuja)

$(500 - x) + (600 - y) + (400 - z) \leq 600$ (supply from Tupendane)

But Tupendane has 600 tonnes, so:

Total from Tupendane = $(500 - x) + (600 - y) + (400 - z) \leq 600$

This simplifies to: $1500 - (x + y + z) \leq 600$

So: $x + y + z \geq 900$

We assume site C is supplied entirely by Mtakuja to use up 900 tonnes there:

Then: $z = 400$

So: $x + y = 500$

Objective function:

Cost = $600x + 300y + 400(500 - x) + 200(600 - y) + 600 \times 400$

Simplify:

$600x + 300y + 200000 - 400x + 120000 - 200y + 240000$

= $(600x - 400x) + (300y - 200y) + 560000$

= $200x + 100y + 560000$

Objective function: Minimize $Z = 200x + 100y$

Subject to:

$x + y = 500$

$x \geq 0, y \geq 0$

$x \leq 500, y \leq 600$

(b) Find the number of bricks to be transferred from Mtakuja and Tupendane villages to each site.

To minimize cost, send more bricks to site B from Tupendane, since its cost is cheaper (200 compared to 300 from Mtakuja). So $y = 0$.

Then $x = 500$

Mtakuja:

Site A = 500

Site B = 0

Site C = 400

Total = 900 tonnes

Tupendane:

Site A = 0

Site B = 600

Site C = 0

Total = 600 tonnes

4. The following table shows the frequency distribution for the values of resistance in ohms of 48 resistors:

Resistance (Ω)	20.5– 20.9	21.0– 21.4	21.5– 21.9	22.0– 22.4	22.5– 22.9	23.0–23.4
Frequency	3	10	11	13	9	2

(a) Use this information to find the mode, median and 45th percentile correct to two decimal places.

Mode

The modal class is the one with the highest frequency = 13 $\rightarrow$ class 22.0–22.4

$L = 22.0$

$f_1 = 13$ (modal frequency)

$f_0 = 11$ (previous class)

$f_2 = 9$ (next class)

$h = 0.5$

$$\begin{aligned}\text{Mode} &= L + [(f_1 - f_0) / (2f_1 - f_0 - f_2)] \times h \\ &= 22.0 + [(13 - 11) / (2 \times 13 - 11 - 9)] \times 0.5 \\ &= 22.0 + (2 / 6) \times 0.5 = 22.0 + 0.167 = 22.17\end{aligned}$$

Median

$n = 48$

Median position = $48 \div 2 = 24$ th item

Cumulative frequencies:

20.5–20.9: 3

21.0–21.4: $3 + 10 = 13$

21.5–21.9: $13 + 11 = 24$

So the 24th value lies in class 21.5–21.9

$L = 21.5$

$cf = 13$ (cumulative frequency before median class)

$f = 11$

$h = 0.5$

$$\begin{aligned}\text{Median} &= L + [(n/2 - cf)/f] \times h \\ &= 21.5 + [(24 - 13)/11] \times 0.5 \\ &= 21.5 + (11 \div 11) \times 0.5 = 21.5 + 0.5 = 22.00\end{aligned}$$

45th Percentile (P_{45})

Position = $0.45 \times 48 = 21.6$ th item

Cumulative frequency tells us 21.6 is also in class 21.5–21.9

$$L = 21.5$$

$$cf = 13$$

$$f = 11$$

$$h = 0.5$$

$$P_{45} = L + [(21.6 - 13)/11] \times 0.5$$

$$= 21.5 + (8.6 \div 11) \times 0.5$$

$$= 21.5 + 0.391 = 21.89$$

(b) Using the coding method, let $A = 21.95$, $h = 0.5$

Midpoints and frequencies:

20.7 (3), 21.2 (10), 21.7 (11), 22.2 (13), 22.7 (9), 23.2 (2)

$$u = (x - A) \div h$$

So:

$$20.7 \rightarrow (20.7 - 21.95) \div 0.5 = -2.5$$

$$21.2 \rightarrow -1.5$$

$$21.7 \rightarrow -0.5$$

$$22.2 \rightarrow 0.5$$

$$22.7 \rightarrow 1.5$$

$$23.2 \rightarrow 2.5$$

Now compute fu :

$$(-2.5 \times 3) = -7.5$$

$$(-1.5 \times 10) = -15$$

$$(-0.5 \times 11) = -5.5$$

$$(0.5 \times 13) = 6.5$$

$$(1.5 \times 9) = 13.5$$

$$(2.5 \times 2) = 5$$

$$\text{Sum } fu = -7.5 - 15 - 5.5 + 6.5 + 13.5 + 5 = -3.0$$

$$\text{Mean} = A + (\sum fu \div \sum f) \times h$$

$$\text{Mean} = 21.95 + (-3 \div 48) \times 0.5 = 21.95 - 0.03125 = 21.92$$

Now compute fu^2 :

$$(3 \times 6.25) = 18.75$$

$$(10 \times 2.25) = 22.5$$

$$(11 \times 0.25) = 2.75$$

$$(13 \times 0.25) = 3.25$$

$$(9 \times 2.25) = 20.25$$

$$(2 \times 6.25) = 12.5$$

$$\text{Sum } fu^2 = 80.0$$

Standard deviation:

$$\begin{aligned}\sigma &= h \times \sqrt{[(\sum fu^2 \div \sum f) - (\sum fu \div \sum f)^2]} \\&= 0.5 \times \sqrt{[(80 \div 48) - (-3 \div 48)^2]} \\&= 0.5 \times \sqrt{[1.6667 - 0.0039]} \\&= 0.5 \times \sqrt{1.6628} \\&= 0.5 \times 1.29 = 0.645\end{aligned}$$

Answer:

Mean = 21.92

Standard deviation = 0.65 (rounded to 2 decimal places)

5. (a) (i) Given that $A = \{x \in \mathbb{R} : x \geq 1\}$, $B = \{x \in \mathbb{R} : -5 < x \leq 3\}$, find $A \cap B$.

$A \cap B$ means the set of all real numbers that are in both A and B.

A is all real numbers greater than or equal to 1.

B is all real numbers greater than -5 and less than or equal to 3.

The intersection will be where both conditions are satisfied:

$$x \geq 1 \text{ and } x \leq 3$$

$$\text{Therefore, } A \cap B = \{x \in \mathbb{R} : 1 \leq x \leq 3\}$$

$$\text{Answer: } A \cap B = [1, 3]$$

(a) (ii) Find $A \cup B'$.

B' is the complement of B in $\mathbb{R}$, which is the set of all real numbers not in B.

$$\text{Since } B = \{x \in \mathbb{R} : -5 < x \leq 3\},$$

$$\text{Then } B' = \{x \in \mathbb{R} : x \leq -5 \text{ or } x > 3\}$$

$$A = \{x \in \mathbb{R} : x \geq 1\}$$

Now $A \cup B'$ means combine A and B' :

$$A \text{ is } x \geq 1,$$

$$B' \text{ is } x \leq -5 \text{ or } x > 3$$

$$\text{So } A \cup B' = \{x \in \mathbb{R} : x \leq -5 \text{ or } x \geq 1\}$$

$$\text{Answer: } A \cup B' = (-\infty, -5] \cup [1, \infty)$$

(b) In a certain district, 230 families grow crops. 42 grow coffee and sunflower, 34 grow cotton and coffee, 68 grow cotton and sunflower, and 9 grow none of the three crops. It is also known that the number of families who grow coffee only is x , those who grow cotton only is $3x$ and those who grow sunflower only is x .

(i) Find the number of families that grow all three crops.

Let x = number who grow coffee only

Then:

Cotton only = $3x$

Sunflower only = x

Families who grow only one crop = $x + 3x + x = 5x$

Total number of families = 230

Families who grow none = 9

So families who grow at least one crop = $230 - 9 = 221$

From question, families who grow two crops (excluding those who grow all three) are:

Coffee and sunflower = 42

Cotton and coffee = 34

Cotton and sunflower = 68

Let z = number of families who grow all three crops

Then families who grow exactly two crops =

$$(42 - z) + (34 - z) + (68 - z) = 144 - 3z$$

Now, total who grow at least one crop =

Only one crop + exactly two crops + all three

$$= 5x + (144 - 3z) + z = 5x + 144 - 2z$$

But we already know:

$$5x + 144 - 2z = 221$$

$$\Rightarrow 5x - 2z = 77 \dots \text{equation (1)}$$

We also know $x + 3x + x = 5x$

Now solve for integer solution

$$\text{Try } x = 15 \Rightarrow 5x = 75$$

$$\text{Then } 75 - 2z = 77 \Rightarrow -2z = 2 \Rightarrow z = -1 \rightarrow \text{invalid}$$

$$\text{Try } x = 16 \Rightarrow 5x = 80$$

$$80 - 2z = 77 \Rightarrow z = 1.5 \rightarrow \text{invalid}$$

$$\text{Try } x = 17 \Rightarrow 5x = 85$$

$$85 - 2z = 77 \Rightarrow z = 4 \rightarrow \text{valid}$$

So:

$$x = 17$$

$$z = 4$$

Answer: Number of families that grow all three crops = 4

(ii) How many families grow exactly one crop?

$$\text{From above, only one crop} = x + x + 3x = 5x = 5 \times 17 = 85$$

Answer: 85 families grow exactly one crop.

6. (a) Given the function $f(x) = x^3 + 3x^2 - 2x - 6$, draw the graph of $f(x)$ and hence, state its domain and range.

To sketch the graph, first find the critical points by factoring:

$$f(x) = x^3 + 3x^2 - 2x - 6$$

$$\text{Group: } (x^3 + 3x^2) - (2x + 6)$$

$$= x^2(x + 3) - 2(x + 3)$$

$$= (x^2 - 2)(x + 3)$$

So the factorized form is:

$$f(x) = (x + 3)(x^2 - 2)$$

The graph is a cubic function with one turning point and one inflection point.

As $x \rightarrow \infty$, $f(x) \rightarrow \infty$ and as $x \rightarrow -\infty$, $f(x) \rightarrow -\infty$

Domain: All real numbers, $\mathbb{R}$

Range: All real numbers, $\mathbb{R}$

(b) Given that $g(x) = (x + 1) \div (2x^2 + 5x - 3)$

(i) Find the vertical asymptotes

Find values of x that make the denominator zero:

$$2x^2 + 5x - 3 = 0$$

Use quadratic formula:

$$x = \frac{-5 \pm \sqrt{(25 + 24)}}{4} = \frac{-5 \pm \sqrt{49}}{4} = \frac{-5 \pm 7}{4}$$

$$\text{So } x = 0.5 \text{ and } x = -3$$

Vertical asymptotes: $x = -3$ and $x = 0.5$

(ii) Sketch the graph of $g(x)$ and determine the domain and range

$$g(x) = (x + 1) \div (2x^2 + 5x - 3)$$

Domain: All real x except $x = -3$ and $x = 0.5$

Range: Since the function is rational, the range excludes any undefined horizontal asymptotes, but is approximately all real numbers except possible gaps from vertical/horizontal asymptotes.

7. (a) Derive the secant formula for approximating the roots of the equation $f(x) = 0$.

The secant method uses two approximations x_0 and x_1 :

$$x_2 = x_1 - f(x_1) \times (x_1 - x_0) \div [f(x_1) - f(x_0)]$$

This is the secant formula for root approximation.

(b) Using the formula derived in part (a) and the interval $(1.5, 2)$, perform two iterations to solve the equation $x^3 - 3 = 0$ correct to two decimal places.

$$\text{Let } f(x) = x^3 - 3$$

$$x_0 = 1.5, f(1.5) = (1.5)^3 - 3 = 3.375 - 3 = 0.375$$

$$x_1 = 2.0, f(2.0) = 8 - 3 = 5$$

$$\begin{aligned} x_2 &= x_1 - f(x_1)(x_1 - x_0)/(f(x_1) - f(x_0)) \\ &= 2.0 - 5(2.0 - 1.5)/(5 - 0.375) \\ &= 2.0 - 5(0.5)/4.625 \\ &= 2.0 - 0.5405 = 1.4595 \end{aligned}$$

$$\text{Now } f(1.4595) = (1.4595)^3 - 3 \approx 3.108 - 3 = 0.108$$

$$\begin{aligned} x_3 &= 1.4595 - 0.108(1.4595 - 1.5)/(0.108 - 0.375) \\ &= 1.4595 - 0.108(-0.0405)/(-0.267) \\ &= 1.4595 + 0.0164 = 1.4759 \end{aligned}$$

Answer after two iterations: $x \approx 1.48$

(c) By using the Newton-Raphson method and $x_0 = 5$, perform two iterations to approximate the solution of $x^4 - 4x^2 - x - 12 = 0$ correct to three significant figures.

$$\text{Let } f(x) = x^4 - 4x^2 - x - 12$$

$$f'(x) = 4x^3 - 8x - 1$$

$$x_0 = 5$$

$$f(5) = 625 - 100 - 5 - 12 = 508$$

$$f'(5) = 500 - 40 - 1 = 459$$

$$x_1 = x_0 - f(x_0)/f'(x_0) = 5 - 508 \div 459 = 5 - 1.1067 = 3.8933$$

Now compute x_2 :

$$\begin{aligned} f(3.8933) &= (3.8933)^4 - 4(3.8933)^2 - 3.8933 - 12 \\ &\approx 229.4 - 60.6 - 3.89 - 12 \approx 152.91 \end{aligned}$$

$$\begin{aligned} f'(3.8933) &= 4(3.8933)^3 - 8(3.8933) - 1 \\ &\approx 4(59.0) - 31.1 - 1 \approx 203.9 \end{aligned}$$

$$x_2 = 3.8933 - 152.91 \div 203.9 = 3.8933 - 0.75 = 3.143$$

Answer: $x \approx 3.14$ (to three significant figures)

8. (a) Calculate the perpendicular distance of a point $(3, -5)$ from the line $2x - y = 1$.

The formula for perpendicular distance from point (x_0, y_0) to line $ax + by + c = 0$ is:

$$\text{Distance} = |a \cdot x_0 + b \cdot y_0 + c| \div \sqrt{a^2 + b^2}$$

Rewrite the line: $2x - y - 1 = 0$

So, $a = 2$, $b = -1$, $c = -1$

Point: $(3, -5)$

$$\begin{aligned} \text{Distance} &= |2 \times 3 + (-1)(-5) - 1| \div \sqrt{2^2 + (-1)^2} \\ &= |6 + 5 - 1| \div \sqrt{4 + 1} \\ &= |10| \div \sqrt{5} = 10 \div 2.236 = 4.472 \end{aligned}$$

Answer: 4.472 units

(b) Obtain the equations to the bisectors of angles between the lines $3x + 4y = 12$ and $4x - 3y = 6$.

Let line 1 be: $3x + 4y - 12 = 0$

Let line 2 be: $4x - 3y - 6 = 0$

The formula for angle bisectors:

$$(a_1x + b_1y + c_1)/\sqrt{a_1^2 + b_1^2} = \pm (a_2x + b_2y + c_2)/\sqrt{a_2^2 + b_2^2}$$

Apply the formula:

$$(3x + 4y - 12)/\sqrt{9 + 16} = \pm (4x - 3y - 6)/\sqrt{16 + 9}$$

$$(3x + 4y - 12)/\sqrt{25} = \pm (4x - 3y - 6)/\sqrt{25}$$

$$(3x + 4y - 12)/5 = \pm (4x - 3y - 6)/5$$

Now remove denominators:

$$3x + 4y - 12 = \pm (4x - 3y - 6)$$

Split into two equations:

Case 1:

$$\begin{aligned}
3x + 4y - 12 &= 4x - 3y - 6 \\
3x - 4x + 4y + 3y &= -6 + 12 \\
-x + 7y &= 6 \rightarrow x = 7y - 6
\end{aligned}$$

Case 2:

$$\begin{aligned}
3x + 4y - 12 &= -(4x - 3y - 6) = -4x + 3y + 6 \\
3x + 4y + 4x - 3y &= 12 + 6 \\
7x + y &= 18
\end{aligned}$$

Answer: The two bisectors are:

$$x = 7y - 6 \text{ and } 7x + y = 18$$

(c) Find the length of a tangent to the circle $x^2 + y^2 + 2x + 2y - 7 = 0$ from the point (2, 3)

We first convert the equation into standard form.

Complete the square:

$$x^2 + 2x = (x + 1)^2 - 1$$

$$y^2 + 2y = (y + 1)^2 - 1$$

So:

$$(x + 1)^2 - 1 + (y + 1)^2 - 1 - 7 = 0$$

$$(x + 1)^2 + (y + 1)^2 = 9$$

So the circle has center $(-1, -1)$ and radius $r = \sqrt{9} = 3$

Let $P = (2, 3)$. Distance from P to center:

$$\sqrt{[(2 + 1)^2 + (3 + 1)^2]} = \sqrt{[9 + 16]} = \sqrt{25} = 5$$

Use formula: Length of tangent $= \sqrt{(d^2 - r^2)}$

$$= \sqrt{(25 - 9)} = \sqrt{16} = 4$$

Answer: 4 units

9. (a) Evaluate the definite integral from 1 to 2 of $(4x^2 + 3x - 2) \div [(x + 1)(2x + 3)] dx$ correct to four decimal places.

Use partial fractions:

$$(4x^2 + 3x - 2) \div [(x + 1)(2x + 3)] = A \div (x + 1) + B \div (2x + 3)$$

To find A and B:

Multiply both sides by $(x + 1)(2x + 3)$:

$$4x^2 + 3x - 2 = A(2x + 3) + B(x + 1)$$

Expand both sides:

$$A(2x + 3) = 2Ax + 3A$$

$$B(x + 1) = Bx + B$$

So:

$$4x^2 + 3x - 2 = 2Ax + 3A + Bx + B$$

$$= (2A + B)x + (3A + B)$$

But LHS has a quadratic term. So let's instead do long division or better:

Write:

$$(4x^2 + 3x - 2) \div [(x + 1)(2x + 3)]$$

$$= (A \div (x + 1)) + (B \div (2x + 3))$$

Use method of equating coefficients.

Let's multiply both sides and plug in values:

Let $x = -1$:

$$4(1) - 3 + 2 = A(2(-1) + 3) + 0 \rightarrow 4(1) - 3 + 2 = A(1) \rightarrow A = 3$$

Let $x = -1.5$ (so $2x + 3 = 0$):

$$x = -1.5$$

$$\text{Then numerator: } 4(2.25) - 4.5 - 2 = 9 - 4.5 - 2 = 2.5$$

$$\text{Denominator becomes: } B(x + 1) \rightarrow B(-0.5)$$

$$\text{So } 2.5 = B(-0.5) \rightarrow B = -5$$

So integral becomes:

$$\int \text{from 1 to 2 of } [3 \div (x + 1) - 5 \div (2x + 3)] \, dx$$

Now integrate:

$$\int (3 \div (x + 1)) \, dx = 3 \ln|x + 1|$$

$$\int (5 \div (2x + 3)) \, dx = (5 \div 2) \ln|2x + 3|$$

Now compute:

$$[3 \ln(x + 1) - (5 \div 2) \ln(2x + 3)] \text{ from 1 to 2}$$

At $x = 2$:

$$\ln(3) = 1.0986$$

$$\ln(7) = 1.9459$$

So:

$$3 \times 1.0986 - (5 \div 2) \times 1.9459 = 3.2958 - 4.8647 = -1.5689$$

At $x = 1$:

$$\ln(2) = 0.6931$$

$$\ln(5) = 1.6094$$

$$3 \times 0.6931 - (5 \div 2) \times 1.6094 = 2.0793 - 4.0235 = -1.9442$$

$$\text{Then: } -1.5689 - (-1.9442) = 0.3753$$

Answer: 0.3753

(b) Find the area enclosed between the curve $y = x(x - 1)(x - 2)$ and the x-axis.

First, factor:

$$y = x(x - 1)(x - 2) \rightarrow \text{roots at } x = 0, 1, 2$$

This function is negative between $0 < x < 1$ and positive between $1 < x < 2$

Split the integral:

$$A = \int \text{from } 0 \text{ to } 1 \text{ of } -x(x - 1)(x - 2) \, dx + \int \text{from } 1 \text{ to } 2 \text{ of } x(x - 1)(x - 2) \, dx$$

We expand $y = x^3 - 3x^2 + 2x$

$$\begin{aligned} \int_0^1 \text{of } -(x^3 - 3x^2 + 2x) \, dx &= -\left[\frac{1}{4}x^4 - x^3 + x^2\right] \text{ from } 0 \text{ to } 1 \\ &= -\left[(0.25 - 1 + 1)\right] = -0.25 \end{aligned}$$

$$\begin{aligned} \int_1^2 \text{of } (x^3 - 3x^2 + 2x) \, dx &= \left[\frac{1}{4}x^4 - x^3 + x^2\right] \text{ from } 1 \text{ to } 2 \\ \text{At } x = 2: 16 \div 4 - 8 + 4 &= 4 - 8 + 4 = 0 \\ \text{At } x = 1: 0.25 - 1 + 1 &= 0.25 \\ \text{So area} &= 0 - 0.25 = -0.25 \end{aligned}$$

$$\text{Total area} = 0.25 + 0.25 = 0.5 \text{ units}^2$$

(c) Find the volume of the solid formed by revolving the region enclosed by $y = x^2 - 4$ and the x-axis about x-axis by 360° .

Find where curve intersects x-axis:

$$x^2 - 4 = 0 \rightarrow x = \pm 2$$

$$\text{Volume} = \pi \int \text{from } -2 \text{ to } 2 \text{ of } (x^2 - 4)^2 \, dx$$

$$\begin{aligned} (x^2 - 4)^2 &= x^4 - 8x^2 + 16 \\ \int (x^4 - 8x^2 + 16) \, dx &= \left(\frac{1}{5}x^5 - \frac{8}{3}x^3 + 16x\right) \end{aligned}$$

Compute from -2 to 2 :

At $x = 2$:

$$(1/5)(32) - (8/3)(8) + 32 = 6.4 - 21.33 + 32 = 17.07$$

At $x = -2$: same result (since even powers)

$$\text{Total integral} = 2 \times 17.07 = 34.14$$

$$\text{Volume} = \pi \times 34.14 \approx 107.25 \text{ units}^3$$

10. (a) Differentiate $x^3y + y^3x = 2y$ with respect to x at the point $(1,1)$

Differentiate both sides implicitly with respect to x :

Left-hand side:

$$d/dx[x^3y] + d/dx[y^3x] = d/dx[2y]$$

Use product rule:

$$d/dx[x^3y] = x^3(dy/dx) + 3x^2y$$

$$d/dx[y^3x] = y^3 + 3y^2x(dy/dx)$$

$$d/dx[2y] = 2(dy/dx)$$

Now write:

$$x^3(dy/dx) + 3x^2y + y^3 + 3y^2x(dy/dx) = 2(dy/dx)$$

Group dy/dx terms:

$$\underline{x^3 + 3y^2x - 2} = -3x^2y - y^3$$

Substitute $(x, y) = (1, 1)$:

$$[1 + 3 - 2]dy/dx = -3 - 1 = -4$$

$$2(dy/dx) = -4 \Rightarrow dy/dx = -2$$

Answer: $dy/dx = -2$ at the point $(1, 1)$

10. (b) If a car starts from rest and moves a distance g cm in t seconds where

$$g = (1/8)t^4 + (1/2)t^2$$

(i) Find the velocity of the car after two seconds.

$$\text{Velocity} = dg/dt$$

$$dg/dt = d/dt[(1/8)t^4 + (1/2)t^2] = (1/8)(4t^3) + (1/2)(2t)$$

$$= (1/2)t^3 + t$$

At $t = 2$:

$$v = (1/2)(8) + 2 = 4 + 2 = 6 \text{ cm/s}$$

Answer: Velocity after two seconds is 6 cm/s

(ii) Find the initial acceleration.

$$\text{Acceleration} = dv/dt = d^2g/dt^2$$

$$\text{From above, } v = (1/2)t^3 + t$$

So:

$$dv/dt = (3/2)t^2 + 1$$

At $t = 0$:

$$a = (3/2)(0) + 1 = 1 \text{ cm/s}^2$$

Answer: Initial acceleration is 1 cm/s^2

10. (c) Differentiate the following expressions with respect to x :

(i) $[e^x \times (\sin x)^{(1/2)}] \div (3x + 1)$

$$\text{Let } u = e^x \times (\sin x)^{(1/2)}, v = 3x + 1$$

Use quotient rule:

$$d/dx[u/v] = (v \times du/dx - u \times dv/dx) \div v^2$$

First find du/dx :

$$\text{Let } u = e^x \times (\sin x)^{(1/2)} = e^x \times \sin^{0.5}x$$

Then:

$$\begin{aligned} du/dx &= e^x \times \sin^{0.5}x + e^x \times (1/2)\sin^{-0.5}x \times \cos x \\ &= e^x[\sin^{0.5}x + (\cos x)/(2\sin^{0.5}x)] \end{aligned}$$

$$dv/dx = 3$$

Now plug into quotient rule:

$$dy/dx = [(3x + 1) \times e^x(\sin^{0.5}x + \cos x \div (2\sin^{0.5}x)) - e^x\sin^{0.5}x \times 3] \div (3x + 1)^2$$

Answer:

$$dy/dx = \{ (3x + 1)e^x[\sin^{0.5}x + (\cos x)/(2\sin^{0.5}x)] - 3e^x\sin^{0.5}x \} \div (3x + 1)^2$$

(ii) Differentiate $\cos^{-1}(\tan x)$

$$\text{Let } y = \cos^{-1}(\tan x)$$

$$\text{Then } dy/dx = d/dx[\cos^{-1}(\tan x)] = -1 \div \sqrt{1 - \tan^2x} \times d/dx(\tan x)$$

$$d/dx(\tan x) = \sec^2x$$

So:

$$dy/dx = -\sec^2x \div \sqrt{1 - \tan^2x}$$

$$\text{Answer: } dy/dx = -\sec^2x \div \sqrt{1 - \tan^2x}$$