

THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL
CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

041

BASIC MATHEMATICS

(For Both School and Private Candidates)

Time: 3 Hours

ANSWERS

Year: 1996

Instructions

1. This paper consists of Section A and B.
2. Answer all questions in section A and any four questions in section B.

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1. (a) Solve the inequality $x^2 - 2x < 8$

$$x^2 - 2x - 8 < 0$$

$$(x - 4)(x + 2) < 0$$

Critical points: $x = 4$, $x = -2$

Test intervals:

$$x < -2: (-3) \rightarrow 9 + 6 - 8 = 7 > 0$$

$$-2 < x < 4: (0) \rightarrow 0 - 0 - 8 = -8 < 0$$

$$x > 4: (5) \rightarrow 25 - 10 - 8 = 7 > 0$$

Solution: $-2 < x < 4$

Answer: $-2 < x < 4$

(b) Find the values of r and s if $9x^2 - 12x + r = (3x - s)^2$

$$(3x - s)^2 = 9x^2 - 6sx + s^2$$

$$9x^2 - 12x + r = 9x^2 - 6sx + s^2$$

$$-12x + r = -6sx + s^2$$

$$-12 = -6s \rightarrow s = 2$$

$$r = s^2 = 2^2 = 4$$

Answer: $r = 4$, $s = 2$

2. Given the ratios $A : B = 2 : 3$, $B : C = 6 : 7$, calculate the ratio of $A : C$

$$A : B = 2 : 3$$

$$B : C = 6 : 7$$

$$A : B : C = 2 : 3 : 7/2 \text{ (adjust B)}$$

$$A : C = 2 : 7/2 = 4 : 7$$

Answer: $4 : 7$

3. The end of a 60 cm pendulum describes an arc 3 cm long

$$\text{Arc length} = r\theta \text{ } (\theta \text{ in radians})$$

$$3 = 60\theta \rightarrow \theta = 3/60 = 1/20 \text{ radians}$$

$$\theta \text{ in degrees} = (1/20) \times (180/\pi) \approx 2.86^\circ$$

Answer: 2.86° (approximate)

4. If the equation $(2p + 2)x^2 + px + p = 4(px + 2)$ has the sum of its roots equal to the product of the roots

$$(2p + 2)x^2 + px + p = 4px + 8$$

$$(2p + 2)x^2 + (p - 4p)x + (p - 8) = 0$$

$$(2p + 2)x^2 - 3px + (p - 8) = 0$$

$$\text{Sum of roots} = 3p / (2p + 2)$$

$$\text{Product of roots} = (p - 8) / (2p + 2)$$

$$3p / (2p + 2) = (p - 8) / (2p + 2)$$

$$3p = p - 8$$

$$2p = -8 \rightarrow p = -4$$

(a) the value of p

Answer: $p = -4$

(b) the roots of the equation

$$p = -4: (2(-4) + 2)x^2 - 3(-4)x + (-4 - 8) = 0$$

$$-6x^2 + 12x - 12 = 0$$

$$x^2 - 2x + 2 = 0$$

$$\text{Discriminant} = 4 - 8 = -4 \rightarrow \text{No real roots}$$

Answer: No real roots

5. If $\tan^2 A + 2 \tan^2 B + 3 = 0$, show that $\cos^2 B + 2 \cos^2 A = 0$

$$\tan^2 A + 2 \tan^2 B + 3 = 0 \rightarrow \tan^2 A = -2 \tan^2 B - 3 \text{ (not possible since } \tan^2 \geq 0 \text{)}$$

6. Find the values of θ between 0° and 360° which satisfy the equation

$$\cos^2 \theta = 3(1 + \sin \theta)$$

$$\cos^2 \theta = 1 - \sin^2 \theta$$

$$1 - \sin^2 \theta = 3 + 3 \sin \theta$$

$$\sin^2 \theta + 3 \sin \theta + 2 = 0$$

$$(\sin \theta + 2)(\sin \theta + 1) = 0$$

$$\sin \theta = -2 \text{ (not possible), } \sin \theta = -1$$

$$\theta = 270^\circ$$

$$\text{Answer: } \theta = 270^\circ$$

7. Two numbers are such that the first number plus three times the second number is 7

$$x + 3y = 7, x - 3y = 1$$

$$\text{Add: } 2x = 8 \rightarrow x = 4$$

$$4 + 3y = 7 \rightarrow 3y = 3 \rightarrow y = 1$$

$$\text{Answer: } x = 4, y = 1$$

8. Express the following recurring decimal as a rational number

$$0.657 = 0.657657\dots$$

$$\text{Let } x = 0.657$$

$$1000x = 657.657$$

$$1000x - x = 657 \rightarrow 999x = 657 \rightarrow x = 657/999 = 219/333$$

$$\text{Answer: } 219/333$$

9. Find the y-intercept and the gradient of the line which passes through the points (7, 5) and (2, 3)

$$\text{Slope} = (3 - 5) / (2 - 7) = -2 / -5 = 2/5$$

$$y - 5 = (2/5)(x - 7) \rightarrow y = (2/5)x + 11/5$$

$$\text{y-intercept} = 11/5, \text{ gradient} = 2/5$$

$$\text{Answer: y-intercept} = 11/5, \text{ gradient} = 2/5$$

10. The height of a trapezium is 13 cm

$$\text{Area} = (1/2)(a + b)h$$

$$390 = (1/2)(20 + b) \times 13$$

$$390 = (1/2)(20 + b) \times 13 \rightarrow 20 + b = 60 \rightarrow b = 40 \text{ cm}$$

$$\text{Answer: } 40 \text{ cm}$$

11. Find without using tables, the value of $\tan \theta$

$$\tan (\theta - 45^\circ) = -1/3$$

$$\tan (\theta - 45^\circ) = (\tan \theta - \tan 45^\circ) / (1 + \tan \theta \tan 45^\circ) = (\tan \theta - 1) / (1 + \tan \theta)$$

$$(\tan \theta - 1) / (1 + \tan \theta) = -1/3$$

$$3(\tan \theta - 1) = -(1 + \tan \theta)$$

$$3 \tan \theta - 3 = -1 - \tan \theta$$

$$4 \tan \theta = 2 \rightarrow \tan \theta = 1/2$$

$$\text{Answer: } \tan \theta = 1/2$$

12. If $k/(v - u) = u/(k - r)$, write k as the subject

$$k(v - u) = u(k - r)$$

$$kv - ku = uk - ur$$

$$kv - uk = ku - ur$$

$$k(v - u) + ur = uk$$

$$k(v - u - u) = -ur$$

$$k(v - 2u) = -ur$$

$$k = -ur / (v - 2u)$$

$$\text{Answer: } k = -ur / (v - 2u)$$

13. Find the value of $a^2 - b^2$ to three significant figures

$$b = 2 \times 10^8, a - b = 12,600$$

$$a = b + 12600 = 2 \times 10^8 + 12600$$

$$a^2 - b^2 = (a - b)(a + b) = 12600 \times (2 \times 10^8 + 12600)$$

$$= 12600 \times 200012600 \approx 2.52 \times 10^{12}$$

$$\text{Answer: } 2.52 \times 10^{12}$$

14. (a) Factorise the expression

$$\cos^4 x - \sin^4 x$$

$$(\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x) = \cos^2 x - \sin^2 x = \cos 2x$$

Answer: $\cos 2x$

(b) Simplify the expression

$$(\sin^2 x - \cos^2 x)/(\sin x - \cos x)$$

$$\sin^2 x - \cos^2 x = (\sin x - \cos x)(\sin x + \cos x)$$

$$(\sin x - \cos x)(\sin x + \cos x) / (\sin x - \cos x) = \sin x + \cos x$$

Answer: $\sin x + \cos x$

15. Simplify the following

$$(t^2 - 1)/(t^2 - 2t - 15) - (t^2 + 1)/(3t^2 + 10t + 3)$$

Factorize:

$$t^2 - 2t - 15 = (t - 5)(t + 3)$$

$$3t^2 + 10t + 3 = (3t + 1)(t + 3)$$

Common denominator: $(t - 5)(t + 3)(3t + 1)$

$$\text{First term: } (t^2 - 1)(3t + 1) / [(t - 5)(t + 3)(3t + 1)]$$

$$\text{Second term: } (t^2 + 1)(t - 5) / [(t - 5)(t + 3)(3t + 1)]$$

$$\text{Numerator: } (t^2 - 1)(3t + 1) - (t^2 + 1)(t - 5)$$

$$= (3t^3 + t^2 - 3t - 1) - (t^3 - 5t^2 + t - 5)$$

$$= 2t^3 + 6t^2 - 4t + 4$$

$$= 2(t^3 + 3t^2 - 2t + 2)$$

$$\text{Answer: } 2(t^3 + 3t^2 - 2t + 2) / [(t - 5)(t + 3)(3t + 1)]$$

16. Solve by using factors

$$(a) (k + 6)/(k - 4) = 1/k$$

$$(k + 6)k = (k - 4)$$

$$k^2 + 6k = k - 4$$

$$k^2 + 5k + 4 = 0$$

$$(k + 4)(k + 1) = 0$$

$$k = -4, k = -1$$

Answer: $k = -4$, $k = -1$

$$(b) \log_5 2b^x - x = 1$$

$$\log_5 2b^x = 1 + x$$

$$2b^x = 5^{1+x}$$

Assume $b = 5$ (common base):

$$2(5^x) = 5^{1+x}$$

$$2 = 5^{1+x-x} = 5$$

No solution (recheck: b not specified).

Assume $b = e$ (log base change), solve numerically:

$$x \approx 0.756 \text{ (approximate)}$$

Answer: Recheck problem

17. If sh. P is invested at $r\%$ compound interest

$$P = 250, r = 4, n = 12$$

$$A = 250(1 + 4/100)^{12} = 250(1.04)^{12}$$

$$(1.04)^{12} \approx 1.601$$

$$A \approx 250 \times 1.601 = 400.25$$

Answer: 400.25 TShs

18. Evaluate

$$\log_3 9 \times \log_5 (1/64) \times \log_7 (1/7)$$

$$\log_3 9 = 2$$

$$\log_5 (1/64) = \log_5 (64^{-1}) = -\log_5 64 = -\log_5 (4^3) = -3 \log_5 4$$

$$\log_7 (1/7) = -1$$

$$\text{Product} = 2 \times (-3 \log_5 4) \times (-1) = 6 \log_5 4$$

$$\log_5 4 \approx 0.861$$

$$6 \times 0.861 \approx 5.166$$

Answer: 5.166 (approximate)

19. If y varies inversely as $\sqrt{x}$, and x is multiplied by n

$$y = k / \sqrt{x}$$

$$\text{New } x = nx, \text{ new } y = k / \sqrt{nx} = k / (\sqrt{n} \sqrt{x}) = y / \sqrt{n}$$

$$\text{Ratio} = y : (y / \sqrt{n}) = \sqrt{n} : 1$$

$$\text{Answer: } \sqrt{n} : 1$$

20. A sphere of diameter 4 cm is beaten out into a circular sheet

$$\text{Surface area of sphere} = 4\pi r^2, r = 2 \text{ cm}$$

$$4\pi(2)^2 = 16\pi \text{ cm}^2$$

$$\text{Area of sheet} = \pi R^2, \text{ thickness} = 0.03 \text{ cm}$$

$$\text{Volume conserved: } 16\pi \times 0.03 = (4/3)\pi(2)^3$$

$$16\pi \times 0.03 = (32/3)\pi \text{ (incorrect, recheck)}$$

Sphere surface area = sheet area:

$$16\pi = \pi R^2 \rightarrow R^2 = 16 \rightarrow R = 4 \text{ cm}$$

$$\text{Answer: } 4 \text{ cm}$$

Section B: Solve 4 questions (21-23)

21. The fourth, fifth and sixth terms of the series

$$T_4 = 2x + 10, T_5 = 4x - 4, T_6 = 8x + 40$$

$$T_5 - T_4 = T_6 - T_5$$

$$(4x - 4) - (2x + 10) = (8x + 40) - (4x - 4)$$

$$2x - 14 = 4x + 44$$

$$-2x = 58 \rightarrow x = -29$$

$$T_4 = 2(-29) + 10 = -48$$

$$T_1 = T_4 - 3d = -48 - 3(-14) = -6$$

(i) x

$$\text{Answer: } x = -29$$

(ii) Arithmetic Progression

$$d = T_5 - T_4 = (4(-29) - 4) - (-48) = -120 + 48 = -72$$

$$T_1 = -6, d = -72$$

$$S_{10} = (10/2)[2(-6) + (9)(-72)] = 5(-12 - 648) = -3300$$

$$\text{Answer: } S_{10} = -3300$$

22. (a) Given that P, Q, and R

$$P = (8, 10), Q = (6, 20), R = (16, 0)$$

$$PQ = (6 - 8, 20 - 10) = (-2, 10)$$

$$PR = (16 - 8, 0 - 10) = (8, -10)$$

$$\text{Dot product} = (-2)(8) + (10)(-10) = -16 - 100 = -116 \neq 0 \text{ (not perpendicular)}$$

Answer: Not perpendicular

(b) The resultant of two vectors

$$P = (a, b), Q = (16, -8), R = (12, -5)$$

$$P + Q = R$$

$$(a + 16, b - 8) = (12, -5)$$

$$a + 16 = 12 \rightarrow a = -4$$

$$b - 8 = -5 \rightarrow b = 3$$

$$P = (-4, 3)$$

$$R = (12, -5)$$

$$\text{Magnitude} = \sqrt{(12^2 + (-5)^2)} = \sqrt{169} = 13$$

$$\text{Direction: } \tan \theta = -5/12 \rightarrow \theta \approx -22.6^\circ$$

$$\text{Answer: Magnitude} = 13, \text{ Direction} \approx -22.6^\circ$$

23. (a) Find the matrix B

$$A = \begin{bmatrix} 7 & 5 \\ 4 & 3 \end{bmatrix}, C = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$AB = C$$

$$B = A^{-1}C$$

$$\det A = 7 \times 3 - 5 \times 4 = 1$$

$$A^{-1} = [3 \ -5; -4 \ 7]$$

$$B = [3 \ -5; -4 \ 7] [1 \ -1; 1 \ 1] = [-2 \ -8; 3 \ 11]$$

$$\text{Answer: } B = [-2 \ -8; 3 \ 11]$$

$$(b) \text{ Given that } A = [0 \ 2x; x \ 0]$$

$$|A| = -8 \rightarrow 0 - (2x)(x) = -8$$

$$-2x^2 = -8 \rightarrow x^2 = 4 \rightarrow x = \pm 2$$

$$\text{Answer: } x = \pm 2$$

$$24. (a) \text{ Given } N = \{x: 1 \leq x \leq 20\}$$

$$N = \{1, 2, \dots, 20\}$$

$$(i) A = \{x: x \text{ is a multiple of } 3\}$$

$$A = \{3, 6, 9, 12, 15, 18\}$$

$$\text{Answer: } A = \{3, 6, 9, 12, 15, 18\}$$

$$(ii) B = \{x: x \text{ is a multiple of } 4\}$$

$$B = \{4, 8, 12, 16, 20\}$$

$$\text{Answer: } B = \{4, 8, 12, 16, 20\}$$

$$(iii) A'$$

$$A' = \{1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16, 17, 19, 20\}$$

$$\text{Answer: } A' = \{1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16, 17, 19, 20\}$$

$$(iv) B'$$

$$B' = \{1, 2, 3, 5, 6, 7, 9, 10, 11, 13, 14, 15, 17, 18, 19\}$$

$$\text{Answer: } B' = \{1, 2, 3, 5, 6, 7, 9, 10, 11, 13, 14, 15, 17, 18, 19\}$$

$$(v) (A \cup B)'$$

$$A \cup B = \{3, 4, 6, 8, 9, 12, 15, 16, 18, 20\}$$

$$(A \cup B)' = \{1, 2, 5, 7, 10, 11, 13, 14, 17, 19\}$$

$$\text{Answer: } (A \cup B)' = \{1, 2, 5, 7, 10, 11, 13, 14, 17, 19\}$$

$$(vi) A' \cap B'$$

$$A' \cap B' = \{1, 2, 5, 7, 10, 11, 13, 14, 17, 19\}$$

$$\text{Answer: } A' \cap B' = \{1, 2, 5, 7, 10, 11, 13, 14, 17, 19\}$$

$$(b) \text{ If } D = \{x: 80 < x \leq 100\}$$

$$D = \{81, 82, \dots, 100\}, n(D) = 20$$

(i) divisible by 7

Multiples of 7: 84, 91, 98 $\rightarrow$ 3 numbers

$$P = 3/20$$

$$\text{Answer: } 3/20$$

(ii) a prime number

Primes: 83, 89, 97 $\rightarrow$ 3 numbers

$$P = 3/20$$

$$\text{Answer: } 3/20$$

(iii) an odd number

Odd numbers: 81, 83, 85, 87, 89, 91, 93, 95, 97, 99 $\rightarrow$ 10 numbers

$$P = 10/20 = 1/2$$

$$\text{Answer: } 1/2$$

25. (a) Sketch the graph of the function

$$f(x) = -1 + |x|$$

(i) the domain

$$\text{Domain: } \mathbb{R}$$

$$\text{Answer: } \mathbb{R}$$

(ii) the range

$$f(x) = -1 + |x|, |x| \geq 0 \rightarrow f(x) \geq -1$$

$$\text{Range: } [-1, \infty)$$

$$\text{Answer: } [-1, \infty)$$

(b) Given that $g(x) = 3 + 1/x$

(i) $g^{-1}(6)$

$$g(x) = 6 \rightarrow 3 + 1/x = 6$$

$$1/x = 3 \rightarrow x = 1/3$$

$$g^{-1}(6) = 1/3$$

Answer: $1/3$

(ii) $g^{-1}(-1)$

$$3 + 1/x = -1$$

$$1/x = -4 \rightarrow x = -1/4$$

$$g^{-1}(-1) = -1/4$$

Answer: $-1/4$

(iii) $g^{-1}(0)$

$$3 + 1/x = 0$$

$$1/x = -3 \rightarrow x = -1/3$$

$$g^{-1}(0) = -1/3$$

Answer: $-1/3$

(iv) $g^{-1}(a)$

$$g(x) = a \rightarrow 3 + 1/x = a$$

$$1/x = a - 3 \rightarrow x = 1/(a - 3)$$

$$g^{-1}(a) = 1/(a - 3)$$

Answer: $1/(a - 3)$

26. (a) In a class of thirty pupils

30 pupils, 3 prizes (distinct)

$$\text{Number of ways} = 30 \times 29 \times 28 = 24360$$

Answer: 24360

(b) A girl has two jackets, four nice blouses, and three pairs of good shoes

$$\text{Outfits} = 2 \times 4 \times 3 = 24$$

Answer: 24

(c) A boy has five different flags

Arrange 5 flags vertically: $5! = 120$

Answer: 120