

THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL
CERTIFICATE OF SECONDARY EDUCATION EXAMINATION
041 BASIC MATHEMATICS

(For Both School and Private Candidates)

Time: 3 Hours

ANSWERS

Year: 1997

Instructions

1. This paper consists of Section A and B.
2. Answer all questions in section A and any four questions in section B.

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1. (a) (i) Given that $M = 4 \times 10^3$, express $M/4$ in standard form

$$M = 4 \times 10^3$$

$$M/4 = (4 \times 10^3) / 4 = 10^3 = 1 \times 10^3$$

Answer: 1×10^3

(ii) Express 0.63 in the form a/b where a and b are integers such that $b \neq 0$

$$0.63 = 63/100$$

Answer: $63/100$

(c) If $\log p = 1.813$ and $\log q = 2.513$, find the value of pq^2

$$\log p = 1.813 \rightarrow p = 10^{1.813} \approx 65$$

$$\log q = 2.513 \rightarrow q = 10^{2.513} \approx 326$$

$$q^2 \approx 326^2 \approx 106276$$

$$pq^2 \approx 65 \times 106276 \approx 6.908 \times 10^6$$

Answer: 6.908×10^6 (approximate)

2. (a) Make x the subject of the formula in the equation

$$y = (cx + d)/(x - 2)$$

$$y(x - 2) = cx + d$$

$$yx - 2y = cx + d$$

$$yx - cx = 2y + d$$

$$x(y - c) = 2y + d$$

$$x = (2y + d) / (y - c)$$

Answer: $x = (2y + d) / (y - c)$

(b) If $a + b = (a^2 - 2b)$, find

(i) $3 + 2$

$$a = 3, b = 2$$

$$3 + 2 = 3^2 - 2(2) = 9 - 4 = 5$$

Answer: 5

(ii) $4 + 2$

$a = 4, b = 2$

$$4 + 2 = 4^2 - 2(2) = 16 - 4 = 12$$

Answer: 12

3. (a) The line passing through the point A(k, 4) and B(3, 2k) is parallel to the line $y = x + 0$

Slope of $y = x + 0$ is 1

$$\text{Slope of AB} = (2k - 4) / (3 - k) = 1$$

$$2k - 4 = 3 - k$$

$$3k = 7 \rightarrow k = 7/3$$

$$A(7/3, 4), B(3, 14/3)$$

Answer: $k = 7/3$

(b) A function f is defined by $f(x) = |x - 2|$

(i) Evaluate $f(3)$

$$f(3) = |3 - 2| = 1$$

Answer: 1

(ii) Find x if $f(x) = 6$

$$|x - 2| = 6$$

$$x - 2 = 6 \rightarrow x = 8$$

$$x - 2 = -6 \rightarrow x = -4$$

Answer: $x = 8, x = -4$

4. (a) Factorise $(1000)^3 - (999)^3$ and hence find the exact value

$$(1000)^3 - (999)^3 = (1000 - 999)(1000^2 + 1000 \times 999 + 999^2)$$

$$= 1 \times (1000000 + 999000 + 998001) = 2997001$$

Answer: 2997001

(b) If $\tan A = 2/3$ where A is an acute angle

(i) $\sin A$

$$\tan A = 2/3 \rightarrow \sin A / \cos A = 2/3$$

$$\sin^2 A + \cos^2 A = 1$$

$$\text{Let } \sin A = 2k, \cos A = 3k$$

$$(2k)^2 + (3k)^2 = 1 \rightarrow 13k^2 = 1 \rightarrow k = 1/\sqrt{13}$$

$$\sin A = 2/\sqrt{13}$$

$$\text{Answer: } 2/\sqrt{13}$$

$$(ii) \cos A$$

$$\cos A = 3/\sqrt{13}$$

$$\text{Answer: } 3/\sqrt{13}$$

$$(c) \text{ Rationalise the denominator of } 2\sqrt{3}/\sqrt{2}$$

$$2\sqrt{3} / \sqrt{2} \times (\sqrt{2} / \sqrt{2}) = 2\sqrt{6} / 2 = \sqrt{6}$$

$$\text{Answer: } \sqrt{6}$$

5. (a) In the figure drawn here under PQ = 135 mm

PQ = 135 mm, PQ shorter than 135 mm $\rightarrow$ contradiction

Assume PQ = 135 mm, PR = 6 cm = 60 mm, QR = 9 cm = 90 mm

$$\Delta PQR: PQ^2 = PR^2 + QR^2 \rightarrow 135^2 = 60^2 + 90^2 \rightarrow 18225 = 12600 \text{ (not right triangle)}$$

Recheck: PQ = 15 cm = 150 mm, PR = 6 cm, QR = 9 cm

$$150^2 = 6^2 + 9^2 \rightarrow 22500 \neq 117 \text{ (not right triangle)}$$

Assume typo: Find PQ if PR = 6 cm, QR = 9 cm, right angle at R

$$PQ = \sqrt{(6^2 + 9^2)} = \sqrt{117} \approx 10.82 \text{ cm}$$

PQ shorter than 10.82 cm $\rightarrow$ impossible (hypotenuse longest)

Answer: Recheck problem

6. (a) The distance between two villages P and Q is 160 km

P at 50 km/h, Q at 60 km/h, meet after x hours

$$50x + 60x = 160$$

$$110x = 160 \rightarrow x = 16/11 \text{ hrs}$$

Distance from P = $50 \times 16/11 = 800/11 \approx 72.73$ km

Answer: 72.73 km from P

(b) Find x, y, and z

$P(x) = x^4 - 6x + 1$ divided by $3x + 6 \rightarrow x = -2$

$P(-2) = (-2)^4 - 6(-2) + 1 = 16 + 12 + 1 = 29$

Recheck: Problem likely from earlier context

Assume x, y, z from vectors or equations (not provided)

Answer: Recheck problem

(c) Find x, y given

$[x; y] = [2 \ -1; 0 \ 5] [2; 6]$

$[2 \ -1; 0 \ 5] [2; 6] = [4 \ -6; 0 + 30] = [-2; 30]$

$x = -2, y = 30$

Answer: $x = -2, y = 30$

7. (a) Given $p = (G, 1)$, $q = (-1, 2)$, $r = (1, 3)$, and $g = (1, 1)$

$p = (G, 1) = (1, 1) \rightarrow G = 1$

Find scalars: $p + q = g \rightarrow (1, 1) + (-1, 2) = (0, 3) \neq (1, 1)$

Recheck: $p + q + r = g \rightarrow (1, 1) + (-1, 2) + (1, 3) = (1, 6) \neq (1, 1)$

Assume: $ap + bq + cr = g$

$a(1, 1) + b(-1, 2) + c(1, 3) = (1, 1)$

$a - b + c = 1$

$a + 2b + 3c = 1$

Need third equation or recheck problem

Answer: Recheck problem

8. (a) If A is the set of prime factors of 42 and B is the set of prime factors of 330

$A = \{2, 3, 7\}$

$B = \{2, 3, 5, 11\}$

$$n(A \cap B) = \{2, 3\} = 2$$

Answer: 2

(b) An integer is selected at random

Integers 50 to 60: 50, 51, ..., 60 (11 numbers)

Prime: 53, 59 $\rightarrow$ 2 numbers

Multiple of 4: 52, 56, 60 $\rightarrow$ 3 numbers

Square: None

$$P(\text{prime}) = 2/11$$

$$P(\text{multiple of 4}) = 3/11$$

$$P(\text{square}) = 0$$

Answer: (i) 2/11, (ii) 3/11, (iii) 0, (iv) 0

9. The line defined by the equation $x + y = 12$ crosses the y-axis at A and meets the line $x = 6$ at B

$$x + y = 12$$

$$A \text{ (y-axis, } x = 0\text{): } 0 + y = 12 \rightarrow A = (0, 12)$$

$$B \text{ (} x = 6\text{): } 6 + y = 12 \rightarrow y = 6 \rightarrow B = (6, 6)$$

$$C = (0, 3), M = \text{midpoint of AB}$$

(a) the co-ordinates of the A, B, and M

$$A = (0, 12), B = (6, 6)$$

$$M = ((0 + 6)/2, (12 + 6)/2) = (3, 9)$$

$$\text{Answer: } A = (0, 12), B = (6, 6), M = (3, 9)$$

(b) the co-ordinates of the line AB

$$\text{Equation of AB: Slope} = (6 - 12) / (6 - 0) = -1$$

$$y - 12 = -1(x - 0) \rightarrow y = -x + 12 \rightarrow x + y = 12$$

$$\text{Answer: } x + y = 12$$

(c) the equation of the line which is perpendicular to the line AB passing through M

$$\text{Slope of AB} = -1, \text{ perpendicular slope} = 1$$

$$M = (3, 9)$$

$$y - 9 = 1(x - 3) \rightarrow y = x + 6$$

$$\text{Answer: } y = x + 6$$

10. (a) If $f(x) = (x + 2)/(x^2 - x - 6)$, find the values of x for which the function is not defined

$$x^2 - x - 6 = (x - 3)(x + 2)$$

$$\text{Not defined when } x^2 - x - 6 = 0 \rightarrow x = 3, x = -2$$

$$\text{Answer: } x = 3, x = -2$$

(b) Given $f(x) = [-3 \text{ when } x < -2; 1 \text{ when } -2 \leq x < 1; 4 \text{ when } x \geq 1]$

(i) Sketch the graph of $f(x)$

(ii) State the domain and range of $f(x)$

Domain: $\mathbb{R}$

Range: $\{-3, 1, 4\}$

Answer: Domain: $\mathbb{R}$, Range: $\{-3, 1, 4\}$

(iii) Is $f(x)$ a one to one function?

$$\text{No, e.g., } f(-3) = f(-4) = -3$$

Answer: No

11. A girl wishes to buy skirts for herself

White skirt: 2400, Coloured skirt: 3000

Budget: 48000

Maximize number of skirts: $2x + 3y \leq 48$

$x \geq 1, y \geq 4$ (at least 1 white, 4 coloured)

Vertices:

$$(1, 4): 1 + 4 = 5$$

$$(12, 4): 12 + 4 = 16$$

$$(6, 12): 6 + 12 = 18$$

$$(1, 15): 1 + 15 = 16$$

Maximum at (6, 12): 18 skirts

Answer: 18 skirts

12. The daily wages of one hundred men are distributed as shown below

Wage in TShs. x 1000 | 3.0 - 3.4 | 3.5 - 3.9 | 4.0 - 4.4 | 4.5 - 4.9 | 5.0 - 5.4 | 5.5 - 5.9 | 6.0 - 6.4 | 6.5 - 6.9

Number of men | 4 | 6 | 10 | 14 | x | 20 | 14 | 6

(a) Find x

Total = 100

$$4 + 6 + 10 + 14 + x + 20 + 14 + 6 = 100$$

$$x = 100 - 74 = 26$$

Answer: $x = 26$

(b) Calculate the daily mean wage of the 100 men

Midpoints: 3.2, 3.7, 4.2, 4.7, 5.2, 5.7, 6.2, 6.7 ($\times 1000$ TShs)

$$\text{Mean} = (4 \times 3.2 + 6 \times 3.7 + 10 \times 4.2 + 14 \times 4.7 + 26 \times 5.2 + 20 \times 5.7 + 14 \times 6.2 + 6 \times 6.7) / 100$$

$$= (12.8 + 22.2 + 42 + 65.8 + 135.2 + 114 + 86.8 + 40.2) / 100$$

$$= 519 / 100 = 5.19 \times 1000 = 5190 \text{ TShs}$$

Answer: 5190 TShs

(c) Draw a histogram to represent this data

13. (a) What is a linear transformation?

A transformation $T: \mathbb{R}^n \rightarrow \mathbb{R}^m$ is linear if $T(u + v) = T(u) + T(v)$ and $T(ku) = kT(u)$ for all vectors u, v and scalar k .

Answer: Defined

(b) Is a translation a linear transformation?

No, a translation $T(x, y) = (x + a, y + b)$ does not satisfy $T(0) = 0$ unless $a = b = 0$.

Answer: No

(c) A translation takes the point (5, 8) to (12, -4)

$$\text{Translation vector: } (12 - 5, -4 - 8) = (7, -12)$$

$$(8, 5) \rightarrow (8 + 7, 5 - 12) = (15, -7)$$

Answer: (15, -7)

(d) A linear transformation T maps the point (x, y) onto (x', y')

$$x' = x + y, y' = -x + 2y$$

$$T = \begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix}$$

$$\text{Determinant} = 1 \times 2 - 1 \times (-1) = 3$$

$$T^{-1} = (1/3) \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$$

$$\text{Answer: } T = \begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix}, T^{-1} = (1/3) \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$$