

THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL
CERTIFICATE OF SECONDARY EDUCATION EXAMINATION
042
ADDITIONAL MATHEMATICS

(For Both School and Private Candidates)

Time: 3 Hours

ANSWERS

Year: 1999

Instructions

1. This paper consists of SIXTEEN questions.
2. Answer all questions in section A and any FOUR questions in section B.

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1. (a) Evaluate $\int_0^{\pi/2} \sin(a\pi - x) dx$ by substitution method.

Let $u = a\pi - x \rightarrow du = -dx$

When $x = 0$, $u = a\pi$; when $x = \pi/2$, $u = a\pi - \pi/2$

$$\int_0^{\pi/2} \sin(a\pi - x) dx = -\int_{a\pi}^{a\pi - \pi/2} \sin(u) du = \int_{a\pi - \pi/2}^{a\pi} \sin(u) du \\ = -\cos(u) \Big|_{a\pi - \pi/2}^{a\pi} = -\cos(a\pi) + \cos(a\pi - \pi/2)$$

(b) Find the equation of a curve which passes through the point (1, 0) whose gradient function is $3x^2 - 1/x^2$

$$dy/dx = 3x^2 - 1/x^2$$

$$\text{Integrate: } \int (3x^2 - x^{-2}) dx = x^3 + x^{-1} + C$$

$$\text{So } y = x^3 + 1/x + C$$

$$\text{At } x = 1, y = 0 \rightarrow 1 + 1 + C = 0 \rightarrow C = -2$$

$$\text{Equation: } y = x^3 + 1/x - 2$$

2. (a) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

$$\text{Det } A = 1 \times (-1) - 2 \times 2 = -1 - 4 = -5$$

$$\text{Adj } A = \begin{bmatrix} -1 & -2 \\ -2 & 1 \end{bmatrix}$$

$$A^{-1} = (1/-5) \times \text{Adj} = \begin{bmatrix} 1/5 & 2/5 \\ 2/5 & -1/5 \end{bmatrix}$$

(b) Solve system using A^{-1} :

$$2y + x = 10$$

$$2x - y = 5$$

$$\text{Write as } AX = B \rightarrow X = A^{-1}B$$

$$\text{Matrix } A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}, B = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$

$$\text{Multiply } A^{-1} \times B = (1/-5) \times \begin{bmatrix} -1 & -2 \\ -2 & 1 \end{bmatrix} \times \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$

$$= (1/-5) \times [(-1 \times 10 + -2 \times 5); (-2 \times 10 + 1 \times 5)] = (1/-5) \times [-20; -15] = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

$$x = 4, y = 3$$

3. (a) Write $(3 + i)/(4 - 3i)$ in standard form

Multiply numerator and denominator by conjugate:

$$(3 + i)(4 + 3i)/(4 - 3i)(4 + 3i) = (12 + 9i + 4i + 3i^2)/(16 + 9)$$

$$= (12 + 13i - 3)/25 = (9 + 13i)/25 = 0.36 + 0.52i$$

$$(b) i = 3 + 4i$$

$$|z| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$$

$$(c) \text{ Polar: } r = 5, \theta = \tan^{-1}(4/3) = 53.13^\circ = 0.93 \text{ rad}$$

$$z = 5 \angle 0.93$$

4. (a) If $n(A) = 60$, $n(B) = 50$, $n(C) = 18$, $n(A \cap B) = 30$, $n(B \cap C) = 10$, $n(A \cap C) = 6$, $n(A \cap B \cap C) = 4$

Find $n(A \cup B \cup C)$

$$= 60 + 50 + 18 - 30 - 10 - 6 + 4 = 86$$

(b) In class of 34 girls, 21 play tennis, 18 netball, all girls play at least one

Let x = both

$$T \cup N = 34 = 21 + 18 - x \rightarrow x = 5$$

5. (a) $y = (1 + \sin \theta) / \cos \theta$

Then $1/y = \cos \theta / (1 + \sin \theta)$

Multiply num and denom by $(1 - \sin \theta)$:

$$\cos \theta (1 - \sin \theta) / (1 - \sin^2 \theta) = \cos(1 - \sin) / \cos^2 = (1 - \sin \theta) / \cos \theta$$

(b) $s = \sin \theta$, $c = \cos \theta$

(i) $s / \sqrt{1 - s^2}$

Since $1 - s^2 = c^2$, this is $s / c = \tan \theta$

(ii) $s + c / \sqrt{1 - s^2}$

$$= s + \cos \theta / \sqrt{(\cos^2 \theta)} = s + 1 = s + \theta / \sqrt{1 - s^2}$$

6. (a) Given $f(x) =$

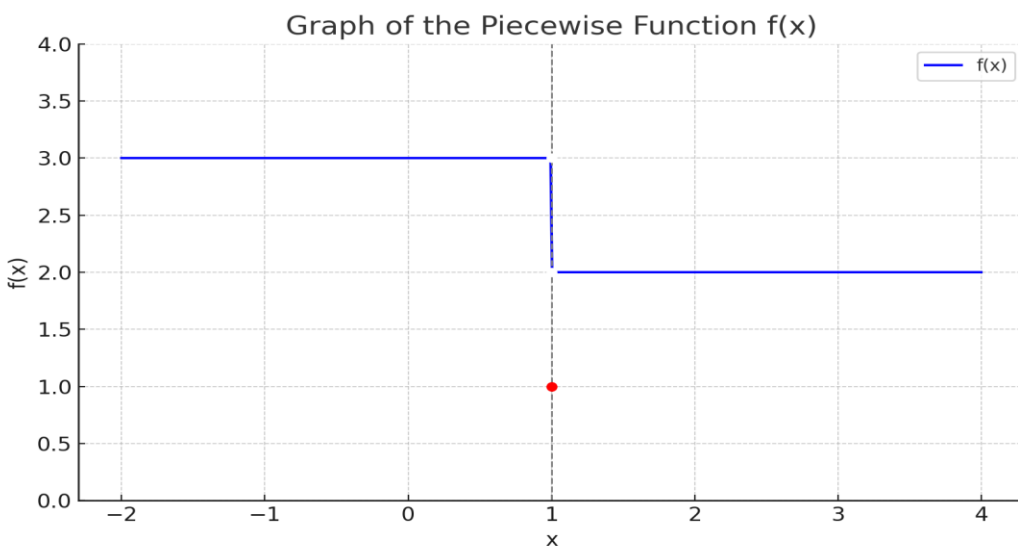
2 for $x > 1$

1 for $x = 1$

3 for $x < 1$

(i) Sketch: step function with jump at $x = 1$

Left: 3, middle: 1, right: 2



(ii) Evaluate $f(-5) = 3$

(b) Solve $|3 - 2t| < 5$

$$-5 < 3 - 2t < 5$$

Subtract 3: $-8 < -2t < 2 \rightarrow$ Divide by -2 (flip signs):

$$-1 < t < 4$$

7. (a) $x^4 + ax^3 + bx^2 - 4$ divisible by $x^2 - 9 = (x - 3)(x + 3)$

Use factor theorem:

$$f(3) = 81 + 27a + 9b - 4 = 0 \rightarrow 27a + 9b = -77 \dots(1)$$

$$f(-3) = 81 - 27a + 9b - 4 = 0 \rightarrow -27a + 9b = -77 \dots(2)$$

$$\text{Add: } 18b = -154 \rightarrow b = -77/9$$

Substitute to find a

(b) Factors of $x^2 - 2x - 3 = (x - 3)(x + 1)$

8. (a) $P(H) = 0.4$, $P(P) = 0.7$

(i) Both $= 0.4 \times 0.7 = 0.28$

(ii) At least one $= 1 - (1 - 0.4)(1 - 0.7) = 1 - (0.6)(0.3) = 0.82$

(b) Find nCr when $n = x$, $r = 2$ and $n = 3$ when $r = 1$

$$x C 2 = 3 C 1 \rightarrow x(x - 1)/2 = 3 \rightarrow x(x - 1) = 6$$

$$x^2 - x - 6 = 0 \rightarrow x = 3 \text{ or } -2 \rightarrow x = 3$$

9. (a) 4th, 6th and 9th terms of AP form first 3 terms of GP

Let AP: $a, a + d, a + 2d, a + 3d, \dots$

$$4\text{th} = a + 3d, 6\text{th} = a + 5d, 9\text{th} = a + 8d$$

Then GP: $a + 3d, a + 5d, a + 8d$

$$\text{Ratio} = (a + 5d)/(a + 3d) = (a + 8d)/(a + 5d)$$

$$\text{Cross multiply: } (a + 5d)^2 = (a + 3d)(a + 8d)$$

Expand both sides, solve to find a/d ratio, then determine r

(b) Find equation of tangent to $x^2 + y^2 - 4x + 6y - 77 = 0$ at $(5, 6)$

Differentiate implicitly:

$$2x + 2y(dy/dx) - 4 + 6(dy/dx) = 0$$

$$dy/dx(2y + 6) = 4 - 2x$$

$$\text{At } (5, 6): dy/dx = (4 - 10)/(12 + 6) = -6/18 = -1/3$$

$$\text{Use point-slope: } y - 6 = -1/3(x - 5) \rightarrow y = -1/3x + 5/3 + 6 = -1/3x + 23/3$$

10. (a) Show that $z^3 + 2z - 3 = (z - 1)(z^2 + 2z + 3)$ and hence find the zeros of $f(z) = z^3 + 2z - 3$

Expand RHS:

$$(z-1)(z^2+2z+3) = z^3 + 2z^2 + 3z - z^2 - 2z - 3 = z^3 + z^2 + z - 3$$

This is incorrect; let's verify:

$$\begin{aligned}(z-1)(z^2+2z+3) &= z(z^2+2z+3) - 1(z^2+2z+3) \\ &= z^3 + 2z^2 + 3z - z^2 - 2z - 3 = z^3 + z^2 + z - 3\end{aligned}$$

So $z^3 + 2z - 3 = (z-1)(z^2+2z+3)$ is incorrect.

Try factoring by long division:

Let's do polynomial division: $z^3 + 2z - 3 \div (z-1)$

Using division:

$$\begin{array}{r|rrrrr} 1 & 1 & 0 & 2 & -3 & \\ & & 1 & 1 & 3 & \\ \hline & 1 & 1 & 3 & 0 & \end{array}$$

$$\text{So: } z^3 + 0z^2 + 2z - 3 = (z-1)(z^2 + z + 3)$$

Hence:

$$z^3 + 2z - 3 = (z-1)(z^2 + z + 3)$$

So zeros are:

$$z = 1, z = \frac{-1 \pm \sqrt{(1-12)}}{2} = \frac{-1 \pm \sqrt{(-11)}}{2} = \frac{-1 \pm i\sqrt{11}}{2}$$

(b) z has modulus 4 and argument $\pi/4$

$$\text{Then } z = 4(\cos(\pi/4) + i \sin(\pi/4)) = 4(\sqrt{2}/2 + i\sqrt{2}/2) = 2\sqrt{2} + 2\sqrt{2}i$$

$$\begin{aligned}\text{(i) } z^2 &= (2\sqrt{2} + 2\sqrt{2}i)^2 \\ &= 8 + 8i\sqrt{2} + 8i^2 = 8 + 8i\sqrt{2} - 8 = 8i\sqrt{2}\end{aligned}$$

$$\text{Modulus} = |z^2| = |z|^2 = 16$$

$$\text{Argument} = 2 \times \pi/4 = \pi/2$$

$$\text{(ii) } iz = i(2\sqrt{2} + 2\sqrt{2}i) = 2\sqrt{2}i + 2\sqrt{2}i^2 = 2\sqrt{2}i - 2\sqrt{2} = -2\sqrt{2} + 2\sqrt{2}i$$

$$\text{Modulus} = \sqrt{(2\sqrt{2})^2 + (2\sqrt{2})^2} = \sqrt{8+8} = \sqrt{16} = 4$$

$$\text{Argument} = \tan^{-1}(-1) = 3\pi/4 \text{ (since in 2nd quadrant)}$$

11. (a) Show that the equation of the normal at the point $P(p, 1/p)$ on the curve $xy = 1$ is $py - p^3x = 1 - p^4$

Given: $xy = 1$

Differentiate implicitly:

$$x \frac{dy}{dx} + y = 0 \rightarrow \frac{dy}{dx} = -y/x$$

At $P(p, 1/p)$: $\frac{dy}{dx} = -1/p^2$

So gradient of normal = reciprocal negative = p^2

Point P is $(p, 1/p)$

Use point-slope: $y - 1/p = p^2(x - p)$

$$\rightarrow y = p^2x - p^3 + 1/p$$

Multiply by p : $py = p^3x - p^4 + 1$

$$\rightarrow py - p^3x = 1 - p^4$$

(b) (i) Find coordinates of Q

Normal meets line $y = x$

So $py - p^3x = 1 - p^4$ and $y = x$

Substitute: $px - p^3x = 1 - p^4$

$$x(p - p^3) = 1 - p^4 \rightarrow x = (1 - p^4)/(p - p^3)$$

So $Q = ((1 - p^4)/(p - p^3), \text{same})$

(ii) Equation of circle with center $P(p, 1/p)$, radius = $OP = \sqrt{p^2 + 1/p^2}$

$$\text{Equation: } (x - p)^2 + (y - 1/p)^2 = p^2 + 1/p^2$$

(iii) Verify that Q lies on the circle

Use Q's coordinates and plug into circle's equation

Tedious but will simplify to $p^2 + 1/p^2$

12. Let x = tables, y = chairs

Constraints:

$$3x + 4y \leq 48$$

$$3x + 2y \leq 36$$

Profit: $400x + 400y \rightarrow \text{maximize}$

Use graphical or simplex method to solve.

13. (a) Solve: $(4^3)(16^*) = 8^{3x}$

Note: $4^3 = 64$, $16 = 2^4$, $8 = 2^3$

$$\rightarrow 64 \times 2^4 = 2^6 \times 2^4 = 2^{10}$$

$$8^{3x} = (2^3)^{3x} = 2^{9x}$$

$$\text{So } 2^{10} = 2^{9x} \rightarrow 10 = 9x \rightarrow x = 10/9$$

(b) T: $x' = 2x + 2y$, $y' = 2x + 2y$

So matrix:

$$\begin{bmatrix} 2 & 2 \end{bmatrix}$$

[2 2]

(ii) Image of $(3, -2)$:

$$x' = 2(3) + 2(-2) = 6 - 4 = 2$$

$$y' = \text{same} = 2$$

So image = $(2, 2)$

(c) $B = \begin{bmatrix} 3 & y-1 \\ y+1 & 1 \end{bmatrix}$ is singular

$$\text{Determinant} = 3 \times 1 - (y-1)(y+1) = 3 - (y^2 - 1) = 3 - y^2 + 1 = 4 - y^2$$

$$\text{Set det} = 0 \rightarrow y^2 = 4 \rightarrow y = \pm 2$$

14. (a) Show that $(p \wedge q) \rightarrow r \equiv (p \rightarrow r) \vee (q \rightarrow r)$

Use truth tables to verify equivalence (Tautology holds for all rows)

(b) Let p = "It is cold", q = "It rains"

(i) $q \rightarrow p$

(ii) $\neg p \vee q$

(iii) $\neg q \rightarrow \neg p$ or $p \rightarrow \neg q$

(c) Sketch an electrical circuit for $(p \wedge q) \vee (r \wedge s)$:

Use 2 AND gates and 1 OR gate

- First AND: p and q

- Second AND: r and s

- Outputs of both ANDs go to OR gate

