

THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL
CERTIFICATE OF SECONDARY EDUCATION EXAMINATION
041 BASIC MATHEMATICS

(For Both School and Private Candidates)

Time: 3 Hours

ANSWERS

Year: 2002

Instructions

1. This paper consists of Section A and B.
2. Answer all questions in section A and any four questions in section B.

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1. (a) By using mathematical tables evaluate the expression $237.8 \times 0.30873 / \sqrt{0.2508}$

$$237.8 \times 0.30873 \approx 73.4100$$

$$\sqrt{0.2508} \approx 0.5008$$

$$73.4100 / 0.5008 \approx 146.5463$$

To four figures: 146.5

Answer: 146.5

(b) Round each of the numbers $x = 2.354$, $y = 4.843$ and $z = 1.759$ to one decimal place then use the results obtained to find the value of A to two significant figures given that $A = x - 3y / z$

$$x \approx 2.4, y \approx 4.8, z \approx 1.8$$

$$A = 2.4 - (3 \times 4.8) / 1.8 = 2.4 - 14.4 / 1.8 = 2.4 - 8 = -5.6$$

To 2 significant figures: -5.6

Answer: -5.6

2. (a) In a class of 30 students, 17 participate in English debate, 12 participate in English debate and sports. Every student is required to participate in at least one of these two events, find the number of students who participate in

(i) English debate only

$$\text{English debate} = 17, \text{English and sports} = 12$$

$$\text{English only} = 17 - 12 = 5$$

Answer: 5

(ii) If $A = \{a, b, c\}$, $B = \{b, c, d\}$ and $C = \{c, d, e\}$, show that $A \cap (B \cap C) = (A \cap B) \cap C$

$$B \cap C = \{c, d\}$$

$$A \cap (B \cap C) = \{a, b, c\} \cap \{c, d\} = \{c\}$$

$$A \cap B = \{b, c\}$$

$$(A \cap B) \cap C = \{b, c\} \cap \{c, d\} = \{c\}$$

Equal

Answer: Shown

3. (a) If $A = \{1, 3, 5\}$, $B = \{2, 7\}$ and $C = \{1, -4\}$ evaluate $|A - 3B + 2C|$

$$A = 1i + 3j + 5k, B = 2i + 7j, C = i - 4k$$

$$3B = 6i + 21j, 2C = 2i - 8k$$

$$A - 3B + 2C = (1 - 6 + 2)i + (3 - 21)j + (5 - 0 - 8)k = -3i - 18j - 3k$$

$$|A - 3B + 2C| = \sqrt{(-3)^2 + (-18)^2 + (-3)^2} = \sqrt{9 + 324 + 9} = \sqrt{342} \approx 18.49$$

Answer: 18.49 (approximate)

(b) If $\cos A = 3/5$, find the values of $\tan A$

$$\cos A = 3/5$$

$$\sin A = \sqrt{1 - (3/5)^2} = \sqrt{16/25} = 4/5$$

$$\tan A = \sin A / \cos A = (4/5) / (3/5) = 4/3$$

Answer: 4/3

4. (a) If $\sin A = -7$, what is $\log_{10} A^2$?

$\sin A$ cannot be -7 (range of sine is $[-1, 1]$).

Assume typo: If $A = -7$

$$\log_{10} A^2 = \log_{10} (-7)^2 = \log_{10} 49 \approx 1.6902$$

Answer: 1.6902 (approximate, recheck problem)

(b) Express as a single logarithm, the expression $1/2 \log x - 7 \log y + \log Z$

$$1/2 \log x - 7 \log y + \log Z = \log (x^{1/2} / y^7) + \log Z = \log (Z x^{1/2} / y^7)$$

Answer: $\log (Z x^{1/2} / y^7)$

5. (a) Prove that the angles in the same segment of a circle are equal

Angles subtended by the same arc at the circumference are equal (circle theorem).

For arc AB, $\angle ACB = \angle ADB$ (same segment).

Answer: Shown

(b) In the figure above ACD is an equilateral triangle and ABCD is a cyclic quadrilateral given that $\angle C = 20^\circ$, find the size of angle EBC

$$ACD \text{ equilateral} \rightarrow \angle ACD = 60^\circ$$

$$\angle C = 20^\circ \text{ (likely } \angle ECB)$$

Cyclic quadrilateral ABCD: $\angle ABD + \angle ACD = 180^\circ \rightarrow \angle ABD = 120^\circ$

$\triangle ECB$: $\angle ECB = 20^\circ$, $\angle BCE = 60^\circ$ (since $\angle ACD = 60^\circ$)

$$\angle EBC = 180^\circ - 20^\circ - 60^\circ = 100^\circ$$

Answer: 100°

6. (a) A and C are shops 120,000 km² in the ratio 2:3:5 respectively. How much will each get?

$$\text{Total ratio} = 2 + 3 + 5 = 10$$

$$A = (2/10) \times 120,000 = 24,000 \text{ km}^2$$

$$B = (3/10) \times 120,000 = 36,000 \text{ km}^2$$

$$C = (5/10) \times 120,000 = 60,000 \text{ km}^2$$

Answer: A: 24,000 km², B: 36,000 km², C: 60,000 km²

(b) An article is sold at TSh 40,500/-. This price includes 30% Value Added Tax (V.A.T)...

$$\text{Price with VAT} = 40,500$$

$$\text{Price before VAT: } 40,500 / 1.3 = 31,153.85$$

Answer: 31,153.85 TSh

7. (a) Find the equation of the line through the point (2, 3) perpendicular to the line whose equation is $4y - 3x + 1 = 0$

$$4y - 3x + 1 = 0 \rightarrow y = (3/4)x - 1/4$$

Slope = $3/4$, perpendicular slope = $-4/3$

$$\text{Equation: } y - 3 = (-4/3)(x - 2) \rightarrow y = (-4/3)x + 8/3 + 3 = (-4/3)x + 17/3$$

Answer: $y = (-4/3)x + 17/3$

(b) Determine the slope of the line $2x - 3y - 4 = 0$

$$2x - 3y - 4 = 0 \rightarrow 3y = 2x - 4 \rightarrow y = (2/3)x - 4/3$$

Slope = $2/3$

Answer: $2/3$

8. (a) The volume of a rectangular box is 1008 cm³. If its length is 14 cm and its breadth is 9 cm, find its height

$$\text{Volume} = l \times b \times h$$

$$1008 = 14 \times 9 \times h$$

$$h = 1008 / (14 \times 9) = 1008 / 126 = 8 \text{ cm}$$

Answer: 8 cm

(b) The curved surface area of a cylinder is 264 cm^2 . If the height of the cylinder is 14 cm, calculate its volume and radius

$$\text{Curved surface area} = 2\pi rh = 264$$

$$2\pi r(14) = 264 \rightarrow r = 264 / (28\pi) \approx 3 \text{ cm } (\pi \approx 3.14)$$

$$\text{Volume} = \pi r^2 h = \pi(3)^2(14) = 126\pi \approx 395.64 \text{ cm}^3$$

Answer: Radius $\approx 3 \text{ cm}$, Volume $\approx 395.64 \text{ cm}^3$

9. (a) Find the solution set of the following inequality and show the solution on separate linear lines

$$2 < x - 3/3 \leq 5$$

$$2 < (x - 3)/3 \leq 5$$

$$6 < x - 3 \leq 15$$

$$9 < x \leq 18$$

Solution: (9, 18]

Answer: (9, 18]

(b) Solve the following system of equations

$$2x + y - z = 3$$

$$x + 2z = 7 + y$$

$$y = x - 5$$

$$\text{then, } 2x + x - 5 - z = 3, 3x - z = 8, \text{ also } x + 2z = 7 + x - 5$$

$$\text{on solving } x = 3, y = -2, z = 1$$

10. A car travelling at a uniform speed of 24 m/s for 10 seconds stops after decelerating for 4 seconds

(a) Draw a velocity-time graph for the motion of the car

(b) Calculate the deceleration of the car

Initial velocity = 24 m/s, final velocity = 0 m/s, time = 4 s

Deceleration = $(0 - 24) / 4 = -6 \text{ m/s}^2$

Magnitude = 6 m/s^2

Answer: 6 m/s^2

(c) From the graph in 10 (a), calculate the total distance travelled by the car

Area under graph:

Constant speed (0 to 10 s): $24 \times 10 = 240 \text{ m}$

Deceleration (10 to 14 s): $(1/2) \times 24 \times 4 = 48 \text{ m}$

Total distance = $240 + 48 = 288 \text{ m}$

Answer: 288 m

11. (a) Draw using the same set of axes the following inequalities

$$2x + y \leq 16$$

$$x + 2y \leq 11$$

$$x + 3y \leq 15$$

$$x \geq 0$$

$$y \geq 0$$

(b) From the graph in (a) find the points which will make the function or relation $f(x, y) = 3x + 5y$ either a maximum or minimum

Vertices:

$$(0, 0): f = 0$$

$$(0, 5): 2x + y = 16 \rightarrow y = 16, x + 2(5) = 11 \rightarrow x = 1 \text{ (not on } 2x + y \leq 16)$$

$$\text{Recheck: } x + 2y = 11, x + 3y = 15 \rightarrow y = 4, x = 3 \rightarrow (3, 4)$$

$$(3, 4): f = 3(3) + 5(4) = 29$$

$$(5, 2): 2x + y = 16 \rightarrow 2(5) + 2 = 12 \text{ (on line), } x + 3y = 15 \rightarrow 5 + 3(2) = 11 \text{ (not on)}$$

$$(8, 0): 2x + y = 16 \rightarrow x = 8, y = 0 \rightarrow f = 24$$

(0, 5): $x + 2y = 11 \rightarrow y = 5.5$ (not on $x + 3y \leq 15$)

Recheck: (0, 5) on $x + 3y = 15 \rightarrow f = 25$

Minimum at (0, 0): 0

Maximum at (3, 4): 29

Answer: Minimum at (0, 0): 0, Maximum at (3, 4): 29

(c) A random sample of 100 students was chosen from a school. Each student's blood pressure was measured...

Blood pressure (mm Hg) | 50-54 | 55-59 | 60-64 | 65-69 | 70-74 | 75-79 | 80-84 | 85-89

Number of students | 1 | 3 | 6 | 8 | 17 | 30 | 25 | 10

(i) Calculate the mean and mode of blood pressure

Mean: Midpoints: 52, 57, 62, 67, 72, 77, 82, 87

Mean = $(1 \times 52 + 3 \times 57 + 6 \times 62 + 8 \times 67 + 17 \times 72 + 30 \times 77 + 25 \times 82 + 10 \times 87) / 100$

= $(52 + 171 + 372 + 536 + 1224 + 2310 + 2050 + 870) / 100 = 7585 / 100 = 75.85$ mm Hg

Mode: 75-79 (highest frequency = 30)

Answer: Mean ≈ 75.85 mm Hg, Mode: 75-79 mm Hg

(ii) Construct the cumulative frequency table and draw the ogive. From the ogive estimate the median blood pressure

Cumulative frequency:

50-54: 1

55-59: 4

60-64: 10

65-69: 18

70-74: 35

75-79: 65

80-84: 90

85-89: 100

Median (50th percentile): Between 70-74 and 75-79

Median ≈ 74.5 mm Hg (approximate from ogive)

Answer: Median ≈ 74.5 mm Hg

(iii) the percentage of students with blood pressure between 67 mmHg and 76 mmHg

67 mmHg: Between 65-69 $\rightarrow$ 18 students

76 mmHg: Between 75-79 $\rightarrow$ 65 students

Percentage = $(65 - 18) / 100 \times 100 = 47\%$

Answer: 47%

13. (a) Calculate the volume and surface area of a sphere of radius 21 cm

Volume = $(4/3)\pi r^3 = (4/3)\pi(21)^3 = (4/3)\pi(9261) = 12348\pi \approx 38792.52 \text{ cm}^3$

Surface area = $4\pi r^2 = 4\pi(21)^2 = 4\pi(441) = 1764\pi \approx 5541.36 \text{ cm}^2$

Answer: Volume $\approx 38792.52 \text{ cm}^3$, Surface area $\approx 5541.36 \text{ cm}^2$

(b) A sphere is cut by a horizontal plane so that the area of the cross section is $81\pi \text{ cm}^2$...

Cross section area = $\pi r^2 = 81\pi \rightarrow r = 9 \text{ cm}$

Sphere radius = 21 cm

Distance from center = $\sqrt{(21^2 - 9^2)} = \sqrt{(441 - 81)} = \sqrt{360} = 6\sqrt{10} \text{ cm}$

Distance from center = 15 cm (recheck)

Radius of new sphere: 15 cm

Answer: Radius = 15 cm

14. (a) Evaluate the following expression

$[2 \ 4] [-3 \ 1] + [2 \ 1 \ -3]$

$[2 \ 4] [-3 \ 1] = [2(-3) + 4(1)] = [-2]$

$[2 \ 1 \ -3]$ (vector addition not possible with scalar)

Recheck: $[2 \ 4] [-3; 1] = [-2]$, add $[2 \ 1 \ -3] \rightarrow$ dimension mismatch

Assume: $[2 \ 4] [-3 \ 1; 2 \ -3] + [2 \ 1 \ -3]$

$= [2(-3) + 4(2) \ 2(1) + 4(-3)] + [2 \ 1 \ -3] = [-6 + 8 \ 2 - 12] + [2 \ 1 \ -3] = [2 \ -10] + [2 \ 1 \ -3]$

Dimension mismatch, recheck problem.

Answer: Recheck problem

(b) Find x and y given that

$$\begin{bmatrix} 4 & 5 \end{bmatrix} \begin{bmatrix} x \end{bmatrix} = \begin{bmatrix} 3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 \end{bmatrix} \begin{bmatrix} y \end{bmatrix} = \begin{bmatrix} 1 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 5; & 2 & 3 \end{bmatrix} \begin{bmatrix} x; y \end{bmatrix} = \begin{bmatrix} 3; 1 \end{bmatrix}$$

$$4x + 5y = 3$$

$$2x + 3y = 1$$

Multiply second by 2: $4x + 6y = 2$

Subtract: $-y = 1 \rightarrow y = -1$

$$4x + 5(-1) = 3 \rightarrow 4x - 5 = 3 \rightarrow 4x = 8 \rightarrow x = 2$$

Answer: $x = 2, y = -1$

(c) The transformation $\begin{bmatrix} y' \end{bmatrix} = \begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} y \end{bmatrix}$ maps the triangle $A(3, 2)$, $B(1, 2)$ and $C(3, 8)$ onto the triangle $A'B'C'$

$$A' = \begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} 3; 2 \end{bmatrix} = \begin{bmatrix} 15 + 4 \end{bmatrix} = \begin{bmatrix} 19 \end{bmatrix} \text{ (recheck)}$$

$$\text{Transformation: } \begin{bmatrix} x' & y' \end{bmatrix} = \begin{bmatrix} 5 & 2; & 1 & 4 \end{bmatrix} \begin{bmatrix} x; y \end{bmatrix}$$

$$A' = \begin{bmatrix} 5 & 2; & 1 & 4 \end{bmatrix} \begin{bmatrix} 3; 2 \end{bmatrix} = \begin{bmatrix} 15 + 4; & 3 + 8 \end{bmatrix} = \begin{bmatrix} 19, 11 \end{bmatrix}$$

$$B' = \begin{bmatrix} 5 & 2; & 1 & 4 \end{bmatrix} \begin{bmatrix} 1; 2 \end{bmatrix} = \begin{bmatrix} 5 + 4; & 1 + 8 \end{bmatrix} = \begin{bmatrix} 9, 9 \end{bmatrix}$$

$$C' = \begin{bmatrix} 5 & 2; & 1 & 4 \end{bmatrix} \begin{bmatrix} 3; 8 \end{bmatrix} = \begin{bmatrix} 15 + 16; & 3 + 32 \end{bmatrix} = \begin{bmatrix} 31, 35 \end{bmatrix}$$

Answer: $A'(19, 11)$, $B'(9, 9)$, $C'(31, 35)$

15. The function f is defined as

$$f(x) = 3x - 5 \text{ when } x \geq 2$$

$$f(x) = 4 \text{ when } -2.5 < x < 2$$

$$f(x) = x - 2 \text{ when } x \leq -2.5$$

(a) Sketch the graph of f

(b) From the graph determine the domain and range of f

Domain: All real numbers

Range: $x \leq -2.5 \rightarrow f(x) = x - 2 \rightarrow (-\infty, -4.5]$, $-2.5 < x < 2 \rightarrow 4$, $x \geq 2 \rightarrow [1, \infty)$

Total range: $(-\infty, -4.5] \cup \{4\} \cup [1, \infty)$

Answer: Domain: $\mathbb{R}$, Range: $(-\infty, -4.5] \cup \{4\} \cup [1, \infty)$

(c) Find the value of (i) $f(-3)$ (ii) $f(0)$ (iii) $f(5)$ (iv) $f(-2.7)$

(i) $f(-3) = -3 - 2 = -5$

(ii) $f(0) = 4$

(iii) $f(5) = 3(5) - 5 = 10$

(iv) $f(-2.7) = 4$

Answer: (i) -5, (ii) 4, (iii) 10, (iv) 4

16. (a) A fair die is tossed once and the number showing up is recorded. What is the probability of an even number greater than two showing up?

Even numbers: 2, 4, 6

Greater than 2: 4, 6

$P(4 \text{ or } 6) = 2/6 = 1/3$

Answer: $1/3$

(b) A box contains 4 white balls and 5 black balls. Two balls are drawn at random from the box

Total balls = 9

(i) white

$P(\text{both white}) = (4/9) \times (3/8) = 12/72 = 1/6$

Answer: $1/6$

(ii) black

$P(\text{both black}) = (5/9) \times (4/8) = 20/72 = 5/18$

Answer: $5/18$

(c) It is known that $C_x^{18} = C_x^{20}$

$C(n, r) = C(n, n-r)$

$$C_x^{18} = C_x^{20} \rightarrow x - 18 = 20 \rightarrow x = 38$$

$$\text{Or } C_x^{18} = C_x^{20} \rightarrow 18 + 20 = x \rightarrow x = 38$$

Answer: $x = 38$