

THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL
CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

041

BASIC MATHEMATICS

(For Both School and Private Candidates)

Time: 3 Hours

ANSWERS

Year: 2004

Instructions

1. This paper consists of Section A and B.
2. Answer all questions in section A and any four questions in section B.

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1. (a) Using mathematical tables, evaluate the expression: $(28.32^2 \times 0.3574) / \sqrt{0.8732}$. Give the answer correct to four decimal places

$$28.32^2 \approx 802.0224$$

$$\sqrt{0.8732} \approx 0.9345$$

$$(802.0224 \times 0.3574) / 0.9345 \approx 286.6428 / 0.9345 \approx 306.7881$$

To 4 decimal places: 306.7881

Answer: 306.7881

(b) If $a = 2.432 \times 10^5$, $b = 7.42 \times 10^4$ and $c = 0.0324 \times 10^{-3}$, find the value of $R = ab - c$ in standard form correct to two decimal places

$$a = 2.432 \times 10^5, b = 7.42 \times 10^4$$

$$ab = (2.432 \times 10^5) \times (7.42 \times 10^4) = (2.432 \times 7.42) \times 10^9 = 18.04544 \times 10^9 = 1.804544 \times 10^{10}$$

$$c = 0.0324 \times 10^{-3} = 3.24 \times 10^{-5}$$

$$R = ab - c = 1.804544 \times 10^{10} - 3.24 \times 10^{-5} \approx 1.804544 \times 10^{10}$$

Standard form, 2 decimal places: 1.80×10^{10}

Answer: 1.80×10^{10}

2. (a) Given that one of the roots of the equation $2x^2 - kx + 1 = 3 - 0$ is 4, find k

$$2x^2 - kx + 1 = 3 \rightarrow 2x^2 - kx - 2 = 0$$

$$x = 4: 2(4)^2 - k(4) - 2 = 0$$

$$32 - 4k - 2 = 0$$

$$30 - 4k = 0$$

$$k = 7.5$$

Answer: 7.5

(b) If $x^2 + ax + 4$ is a perfect square, find the value of a

Perfect square: $b^2 = 4ac$, $a = 1$, $b = a$, $c = 4$ then,

$$a^2 = 4(1)(4)$$

$$a = 4$$

3. (a) Given the vectors $a = 5i - j$, $b = 3i + j$, $c = 2i - 3j$, calculate the resultant of $a + b - c$

$$a + b - c = (5i - j) + (3i + j) - (2i - 3j) = (5 + 3 - 2)i + (-1 + 1 + 3)j = 6i + 3j$$

Answer: $6i + 3j$

(b) Given that $\sin A = 3/5$, find the values of:

(i) $\cos A$

$$\sin A = 3/5$$

$$\cos^2 A = 1 - \sin^2 A = 1 - (3/5)^2 = 1 - 9/25 = 16/25$$

$$\cos A = \pm 4/5 \text{ (A likely acute)}$$

$$\cos A = 4/5$$

Answer: $4/5$

(ii) $\tan A / (1 + \cos A)$

$$\tan A = \sin A / \cos A = (3/5) / (4/5) = 3/4$$

$$1 + \cos A = 1 + 4/5 = 9/5$$

$$\tan A / (1 + \cos A) = (3/4) / (9/5) = (3/4) \times (5/9) = 15/36 = 5/12$$

Answer: $5/12$

4. (a) Simplify $2 \log_{10} 25 - 3 \log_{10} 5 + \log_{10} 20$

$$2 \log_{10} 25 - 3 \log_{10} 5 + \log_{10} 20$$

$$= \log_{10} 25^2 - \log_{10} 5^3 + \log_{10} 20$$

$$= \log_{10} 625 - \log_{10} 125 + \log_{10} 20$$

$$= \log_{10} (625 \times 20 / 125) = \log_{10} (100) = 2$$

Answer: 2

(b) Find the value of t in the equation $3^{t+1} = 6$

$$3^{t+1} = 6$$

$$t + 1 = \log_3 6$$

$$t = \log_3 6 - 1 \approx 1.6309 - 1 \approx 0.6309$$

Answer: 0.6309 (approximate)

(c) Use the substitution $y = 2^t$ to solve the equation $2^{t+1} - 2^{t+1} + 1 = 2^t$

$$y = 2^t$$

$$2^{t+1} = 2 \times 2^t = 2y$$

$$\text{Equation: } 2y - 2y + 1 = y$$

$$1 = y$$

$$y = 1 \rightarrow 2^t = 1 \rightarrow t = 0$$

Answer: $t = 0$

5. (a) In figure 1 below, the area of the shaded part DEA is 15 m^2 . If $DA = 3 \text{ m}$ and $AB = 6 \text{ m}$, find the area of the quadrilateral BCDE

$$\text{Area of triangle DEA} = 15 \text{ m}^2$$

$$BA = \text{height} = 3 \text{ m}, AB = \text{base} = 6 \text{ m}$$

$$\text{Area of triangle DAB} = \left(\frac{1}{2}\right) \times 6 \times 3 = 9 \text{ m}^2$$

(c) In the circle ABCD below (figure 2), AB is an arc of 44° and CD is an arc of 25° . O is the centre of the circle. What is the degree measure of $\angle DLC$?

$$\text{Arc AB} = 44^\circ, \text{Arc CD} = 25^\circ$$

$$\angle AOB = 44^\circ, \angle COD = 25^\circ$$

$$\angle DLC = \angle DOC \text{ (same arc DC)}$$

$$\angle DOC = 25^\circ$$

Answer: 25°

6. (a) The price of a TV set which includes V.A.T. is shs 133,800,000. If the rate of V.A.T. is 30%, find the price of the TV before V.A.T. was added

$$\text{Let price before VAT} = P$$

$$P + 0.3P = 133,800,000$$

$$1.3P = 133,800,000$$

$$P = 133,800,000 / 1.3 = 102,923,076.92$$

Answer: 102,923,076.92 shs

(b) The second term of an A.P. is 2 and the sixth term is -14. What is the

(i) first term

$$T_2 = a + d = 2$$

$$T_6 = a + 5d = -14$$

$$\text{Subtract: } 4d = -16$$

$$d = -4$$

$$a + (-4) = 2$$

$$a = 6$$

Answer: 6

(ii) common difference?

$$d = -4$$

Answer: -4

7. The coordinates of the points A, B and C are (4, 3), (-3, 7) and (7, -1) respectively. From this information find the:

(a) length of AB

$$A(4, 3), B(-3, 7)$$

$$AB = \sqrt{((-3 - 4)^2 + (7 - 3)^2)} = \sqrt{((-7)^2 + 4^2)} = \sqrt{(49 + 16)} = \sqrt{65}$$

Answer: $\sqrt{65}$

(b) equation of the line BC (Write your answer in the form $ax + by + c = 0$)

$$B(-3, 7), C(7, -1)$$

$$\text{Slope} = (-1 - 7) / (7 - (-3)) = -8 / 10 = -4/5$$

$$\text{Equation: } y - 7 = (-4/5)(x + 3)$$

$$y - 7 = (-4/5)x - 12/5$$

$$y = (-4/5)x - 12/5 + 7 = (-4/5)x + 23/5$$

$$(4/5)x + y - 23/5 = 0$$

$$4x + 5y - 23 = 0$$

Answer: $4x + 5y - 23 = 0$

8. (a) The ratio of the areas of two circles is 50:72. If the radius of the smaller circle is 15 cm, find the radius of the larger circle

$$\text{Area ratio} = 50/72 = 25/36$$

$$\text{Radius ratio} = \sqrt{(25/36)} = 5/6$$

$$\text{Smaller radius} = 15 \text{ cm}$$

$$\text{Larger radius} = 15 \times (6/5) = 18 \text{ cm}$$

Answer: 18 cm

(b) Find the total surface area of a right circular cone whose height is 24 cm and slant height is 25 cm

$$\text{Slant height } l = 25 \text{ cm, height } h = 24 \text{ cm}$$

$$\text{Radius } r = \sqrt{(l^2 - h^2)} = \sqrt{(25^2 - 24^2)} = \sqrt{(625 - 576)} = \sqrt{49} = 7 \text{ cm}$$

$$\text{Total surface area} = \pi r^2 + \pi r l = \pi(7)^2 + \pi(7)(25) = 49\pi + 175\pi = 224\pi$$

$$\pi \approx 3.14: 224 \times 3.14 \approx 703.36 \text{ cm}^2$$

Answer: 703.36 cm²

9. (a) Given that $(a + 1/a)^2 = 14$, find the value of $a^2 + 1/a^2$

$$(a + 1/a)^2 = 14$$

$$a^2 + 2 + 1/a^2 = 14$$

$$a^2 + 1/a^2 = 12$$

Answer: 12

(b) Make q the subject of the equation $pq + x = c(p + q^2)$

$$pq + x = c(p + q^2)$$

$$pq + x = cp + cq^2$$

$$pq - cp - cq^2 + x = 0$$

$$p(q - c) - cq^2 + x = 0$$

$$-cq^2 + p(q - c) + x = 0$$

$$cq^2 - p(q - c) - x = 0$$

$$cq^2 - pq + cp - x = 0$$

$$q^2 - (p/c)q + (cp - x)/c = 0$$

$$\text{Solve for } q: q = [p/c \pm \sqrt{(p/c)^2 - 4(1/c)(cp - x)}] / 2$$

$$\text{Simplify: } q = [p \pm \sqrt{p^2 - 4c(cp - x)}] / (2c)$$

$$\text{Answer: } q = [p \pm \sqrt{p^2 - 4c(cp - x)}] / (2c)$$

(c) Using figure 3 below, show that $\cos^2 \theta + \sin^2 \theta = 1$

Triangle ABC:

$$a = BC, b = AC, c = AB$$

$$\cos \theta = a/c, \sin \theta = b/c$$

$$\cos^2 \theta + \sin^2 \theta = (a/c)^2 + (b/c)^2 = (a^2 + b^2) / c^2$$

$$\text{By Pythagoras: } a^2 + b^2 = c^2$$

$$(a^2 + b^2) / c^2 = c^2 / c^2 = 1$$

$$\text{Thus, } \cos^2 \theta + \sin^2 \theta = 1$$

Answer: Shown

10. (a) Solve the equation $3x - 5 = (5x - 3) / x$ by factorisation

$$3x - 5 = (5x - 3) / x$$

$$x(3x - 5) = 5x - 3$$

$$3x^2 - 5x = 5x - 3$$

$$3x^2 - 10x + 3 = 0$$

$$(3x - 1)(x - 3) = 0$$

$$x = 1/3 \text{ or } x = 3$$

$$\text{Answer: } x = 1/3, x = 3$$

(b) Figure 4 below shows the velocity time graph for a train journey which takes 16 minutes

(i) Calculate the acceleration in m/s^2 during those parts of the journey corresponding to OP and PQ

OP: 0 to 4 min, 0 to 72 km/h

$$72 \text{ km/h} = 72 \times (5/18) = 20 \text{ m/s}$$

$$4 \text{ min} = 240 \text{ s}$$

$$\text{Acceleration} = (20 - 0) / 240 = 1/12 \text{ m/s}^2$$

PQ: 4 to 10 min, 72 to 90 km/h

$$90 \text{ km/h} = 25 \text{ m/s}$$

$$6 \text{ min} = 360 \text{ s}$$

$$\text{Acceleration} = (25 - 20) / 360 = 5/360 = 1/72 \text{ m/s}^2$$

Answer: OP: $1/12 \text{ m/s}^2$, PQ: $1/72 \text{ m/s}^2$

(ii) total length of the journey in metres

Convert all to m/s:

$$\text{OP: (0 to 20 m/s in 240 s)} \rightarrow \text{Area} = (1/2) \times 240 \times 20 = 2400 \text{ m}$$

$$\text{PQ: (20 to 25 m/s in 360 s)} \rightarrow \text{Area} = (20 + 25)/2 \times 360 = 8100 \text{ m}$$

$$\text{QR: (25 m/s for 3 min = 180 s)} \rightarrow \text{Area} = 25 \times 180 = 4500 \text{ m}$$

$$\text{RS: (25 to 0 m/s in 3 min = 180 s)} \rightarrow \text{Area} = (1/2) \times 25 \times 180 = 2250 \text{ m}$$

$$\text{Total} = 2400 + 8100 + 4500 + 2250 = 17250 \text{ m}$$

Answer: 17250 m

11. Two types of food, A and B, contain 4 and 6 units of protein, and 5 and 3 units of starch per kg respectively...

Let x = kg of food A, y = kg of food B

Minimize cost: $C = 2500x + 2000y$

$$\text{Protein: } 4x + 6y \geq 48 \rightarrow 2x + 3y \geq 24$$

$$\text{Starch: } 5x + 3y \geq 45$$

$$x \geq 0, y \geq 0$$

Vertices:

$$(0, 15): 5x + 3y = 45 \rightarrow y = 15, 2x + 3(15) \geq 24 \text{ (satisfied)} \rightarrow C = 2000(15) = 30000$$

$$(12, 0): 2x + 3y = 24 \rightarrow x = 12, 5(12) + 3(0) \geq 45 \text{ (satisfied)} \rightarrow C = 2500(12) = 30000$$

$$(6, 5): 2x + 3y = 24, 5x + 3y = 45 \rightarrow x = 6, y = 5 \rightarrow C = 2500(6) + 2000(5) = 25000$$

Minimum cost at (6, 5): 6 kg of A, 5 kg of B

Answer: 6 kg of A, 5 kg of B

12. (a) The data below shows test scores of a certain class in mathematics:

21 24 22 21 24 23 24 25 26 27 27

24 24 21 22 24 25 26 27 27

(b) Construct a frequency distribution table showing scores and frequency

Score	Frequency
21	3
22	2
23	1
24	6
25	2
26	2
27	4

(c) The sum of the first six terms of an AP is 72 and the second term is seven times the fifth term

(i) Find the first term and the common difference

$$S_6 = (6/2)[2a + (6-1)d] = 3(2a + 5d) = 72 \rightarrow 2a + 5d = 24$$

$$T_2 = a + d, T_5 = a + 4d$$

$$a + d = 7(a + 4d)$$

$$-6a - 27d = 0 \rightarrow 2a + 9d = 0$$

Solve with $2a + 5d = 24$:

$$4d = -24 \rightarrow d = -6$$

$$2a + 9(-6) = 0 \rightarrow a = 27$$

Answer: $a = 27$, $d = -6$

13. In the diagram below, (figure 5), VABCD is a pyramid whose base ABCD is a square with sides 6 cm...

(a) Calculate the

(i) length VA

$$VA = VN = 3 \text{ cm (vertical)}$$

Answer: 3 cm

(ii) angle between the line VA and the plane ABCD

$$VA \perp ABCD \rightarrow \text{Angle} = 90^\circ$$

Answer: 90°

(iii) volume of the pyramid

$$\text{Base area} = 6 \times 6 = 36 \text{ cm}^2$$

$$\text{Height} = VN = 3 \text{ cm}$$

$$\text{Volume} = (1/3) \times 36 \times 3 = 36 \text{ cm}^3$$

Answer: 36 cm^3

14. (a) Find the image of a line $5x + 10y + 9 = 0$ under a reflection in the line $y = x$

$$\text{Reflection over } y = x: (x, y) \rightarrow (y, x)$$

$$5x + 10y + 9 = 0$$

$$\text{Swap } x \text{ and } y: 5y + 10x + 9 = 0$$

$$10x + 5y + 9 = 0$$

$$\text{Answer: } 10x + 5y + 9 = 0$$

(b) Use the matrix which changes vector $r = (4, 18.24)$ to $r' = (18.24, 4)$

$$\text{Reflection over } y = x: \text{Matrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$r = [4; 18.24] \rightarrow r' = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} [4; 18.24] = [18.24; 4]$$

$$\text{Answer: Matrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

(c) If $M = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$, where $\theta = 135^\circ$, find $M(\theta)$

$$\theta = 135^\circ$$

$$\cos 2\theta = \cos 270^\circ = 0$$

$$\sin 2\theta = \sin 270^\circ = -1$$

$$M = [0 \ -1; \ -1 \ 0]$$

$$\text{Answer: } [0 \ -1; \ -1 \ 0]$$

15. (a) The function f is defined by

$$f(x) = |x + 5| \text{ for } -5 < x < 0$$

$$f(x) = x^2 \text{ for } 0 \leq x < 3$$

(i) Find $f(-4)$

$$f(-4) = |-4 + 5| = 1$$

Answer: 1

(ii) Give the domain and range of $f(x)$

Domain: $(-5, 3)$

Range: $(-5, 0) \rightarrow (0, 5)$, $[0, 3) \rightarrow [0, 9)$

Total range: $[0, 9)$

Answer: Domain: $(-5, 3)$, Range: $[0, 9)$

(iii) Sketch the graph of $f(x)$

15. (c) With reference to figures 6 and 7 below

(i) the values of length y and angle x

Figure 6:

$$\sin 32^\circ = y / 12$$

$$y = 12 \sin 32^\circ \approx 12 \times 0.5299 \approx 6.36 \text{ cm}$$

Figure 7:

$$\tan x = 3 / 4$$

$$x = \tan^{-1}(3/4) \approx 36.87^\circ$$

Answer: $y \approx 6.36 \text{ cm}$, $x \approx 36.87^\circ$

16. (a) Two numbers are chosen at random from 1, 2 and 3. What is the probability that their sum is odd?

Pairs: (1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)

Total = 9

Odd sum: (1, 2), (2, 1), (1, 3), (3, 1), (2, 3), (3, 2) → 6 pairs

$$P(\text{odd}) = 6/9 = 2/3$$

Answer: $2/3$

(b) If A and B are two events such that $P(A) = 1/5$, $P(B) = 1/2$ and $P(A \cap B) = 1/10$, find

(i) $P(A \cup B)$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 1/5 + 1/2 - 1/10 = 3/5$$

Answer: $3/5$

(ii) $P(A \cap B')$

$$P(A \cap B') = P(A) - P(A \cap B) = 1/5 - 1/10 = 1/10$$

Answer: $1/10$

(c) At a second hand car show, 20% of the cars have no engine, 40% have bad tyres and 15% have no engine and have bad tyres...

No engine = 20%, Bad tyres = 40%, No engine and bad tyres = 15%

$$\text{Good tyres and engine} = 100\% - (20\% + 40\% - 15\%) = 55\%$$

$$P(\text{good tyres and engine}) = 0.55$$

Answer: 0.55