

THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL
CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

041

BASIC MATHEMATICS

(For Both School and Private Candidates)

Time: 3 Hours

ANSWERS

Year: 2013

Instructions

1. This paper consists of Section A and B.
2. Answer all questions in section A and any four questions in section B.

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1. (a) A shopkeeper sold 292 t-shirts at the price of sh. 6950. Estimate how much money she got.

Answer:

$$292 \times 6950$$

$$\text{Round: } 300 \times 7000 = 2100000$$

Final Answer: sh. 2100000

(b) Express 0.9% in the form $\frac{1}{n}$ where a and b are integers, such that $b \neq 0$.

Answer:

$$0.9\% = 0.9/100 = 9/1000$$

$$\frac{1}{n} \text{ form: } 9/1000 = 1/(1000/9) = 1/(1000/9)$$

Final Answer: $1/(1000/9)$

2. (a) If $\log y^2 - 2 \log (3x + 1) = 1$, express y in terms of x.

Answer:

$$\log y^2 - 2 \log (3x + 1) = 1$$

$$\log y^2 - \log (3x + 1)^2 = 1$$

$$\log (y^2/(3x + 1)^2) = 1$$

$$y^2/(3x + 1)^2 = 10^1 = 10$$

$$y^2 = 10 (3x + 1)^2$$

$$y = \pm \sqrt{10 (3x + 1)^2} = \pm (3x + 1) \sqrt{10}$$

Final Answer: $y = \pm (3x + 1) \sqrt{10}$

(b) Simplify $\sqrt[4]{7/\sqrt{3}}$ by rationalizing the denominator.

Answer:

$$\sqrt[4]{7/\sqrt{3}} = \sqrt[4]{7} / \sqrt[4]{\sqrt{3}} = \sqrt[4]{7} / 3^{1/4}$$

This interpretation is complex; assume $\sqrt[4]{7/\sqrt{3}}$ means $(\sqrt{7})/\sqrt{3}$:

$$(\sqrt{7})/\sqrt{3} \times (\sqrt{3}/\sqrt{3}) = (\sqrt{7} \times \sqrt{3})/(\sqrt{3} \times \sqrt{3}) = \sqrt{21}/3$$

Final Answer: $\sqrt{21}/3$

3. (a) If $m * n = m + n - 4n$, find x given that $3 * (x + 1) = -27$.

Answer:

$$m * n = m + n - 4n = m - 3n$$

$$3 * (x + 1) = 3 - 3(x + 1) = 3 - 3x - 3 = -3x$$

$$-3x = -27$$

$$x = 27/3 = 9$$

Final Answer: $x = 9$

(b) There are 48 men at a meeting of whom 24 are teachers, 36 are parents and 16 are both teachers and parents. By using a Venn diagram, find the number of men who are neither teachers nor parents.

Answer:

$$\text{Total men} = 48$$

$$\text{Teachers (T)} = 24$$

$$\text{Parents (P)} = 36$$

$$T \cap P = 16$$

$$T \text{ only} = 24 - 16 = 8$$

$$P \text{ only} = 36 - 16 = 20$$

$$T \cup P = 8 + 16 + 20 = 44$$

$$\text{Neither} = 48 - 44 = 4$$

Final Answer: 4

4. (a) Find the direction cosines of $\angle C = 90^\circ + 12^\circ$, hence show that the sum of the squares of these direction cosines is one.

Answer:

$$\angle C = 90^\circ + 12^\circ = 102^\circ$$

Direction cosines: $\cos C$, $\sin C$

$$\cos 102^\circ = \cos (90^\circ + 12^\circ) = -\sin 12^\circ$$

$$\sin 102^\circ = \sin (90^\circ + 12^\circ) = \cos 12^\circ$$

$$\text{Sum of squares: } (\cos 102^\circ)^2 + (\sin 102^\circ)^2 = (-\sin 12^\circ)^2 + (\cos 12^\circ)^2 = \sin^2 12^\circ + \cos^2 12^\circ = 1$$

Final Answer: Direction cosines: $-\sin 12^\circ$, $\cos 12^\circ$, sum of squares = 1

(b) Find the equation of the line through the points (4, 6) the midpoint of (2, 4) and (10, 4).

Answer:

Midpoint of (2, 4) and (10, 4) = $((2+10)/2, (4+4)/2) = (6, 4)$

Points: (4, 6) and (6, 4)

Slope = $(4 - 6)/(6 - 4) = -2/2 = -1$

Equation using (4, 6):

$$y - 6 = -1(x - 4)$$

$$y - 6 = -x + 4$$

$$y = -x + 10$$

Final Answer: $y = -x + 10$

5. (a) The length of two sides of a triangle are 16 cm and 20 cm. Find the area of the triangle if the included angle is 30° .

Answer:

$$\text{Area} = (1/2) \times a \times b \times \sin \theta$$

$$= (1/2) \times 16 \times 20 \times \sin 30^\circ$$

$$= (1/2) \times 16 \times 20 \times (1/2)$$

$$= 80 \text{ cm}^2$$

Final Answer: 80 cm^2

(b) In the figure below, calculate the length BD.

Answer:

In $\triangle ABC$:

AC = 6 cm, BC = 18 cm, AD = 12 cm, AE = 20 cm

BD = ?

$\triangle ABC$: Use Law of Cosines to find $\angle C$ (but need BD for that, so use similar triangles or coordinates).

Assume $\triangle ABD$ and $\triangle CBE$ are related, but let's use geometric insight:

$\triangle ADE$: $AD = 12$ cm, $AE = 20$ cm, $DE = 16$ cm (Pythagorean triple: $12^2 + 16^2 = 20^2$)

Consider $\triangle ABD$ and $\triangle CBE$:

Need more angles or use coordinate geometry for exact BD. Assume D splits BC proportionally or use area ratios, but figure-based:

Approximate via similar triangles or coordinates (place A at (0,0), C at (6,0), B at (18,0), D at (x,0), E at (x+16,0)).

This requires solving via coordinates or additional angles, which are not given.

6. (a) A bus travels 240km using 16 litres of diesel. How many litres of diesel are needed to drive 900km?

Answer:

Diesel per km = $16/240 = 1/15$ litres/km

For 900 km: $900 \times (1/15) = 60$ litres

Final Answer: 60 litres

(b) If y^2 varies directly to $x - 1$ and inversely to $x + d$ and if $x = 2$, $d = 4$ for $y = 1$, then find x when $y = 2$ and $d = 1$.

Answer:

$$y^2 \propto (x - 1)/(x + d)$$

$$y^2 = k (x - 1)/(x + d)$$

When $x = 2$, $d = 4$, $y = 1$:

$$1^2 = k (2 - 1)/(2 + 4) = k (1/6)$$

$$k = 6$$

When $y = 2$, $d = 1$:

$$2^2 = 6 (x - 1)/(x + 1)$$

$$4 = 6 (x - 1)/(x + 1)$$

$$4(x + 1) = 6(x - 1)$$

$$4x + 4 = 6x - 6$$

$$10 = 2x$$

$$x = 5$$

Final Answer: $x = 5$

7. (a) A radio is bought for sh 400,000 and sold for sh 500,000. Find:

(i) The profit made

(ii) The percentage profit.

Answer:

$$(i) \text{ Profit} = 500000 - 400000 = 100000$$

$$(ii) \text{ Percentage profit} = (100000/400000) \times 100 = 25\%$$

Final Answer: (i) sh 100000, (ii) 25%

(b) Find the time in which sh 300,000 will earn an interest of sh 60,000 if the interest rate is 10% per annum.

Answer:

$$\text{Simple Interest} = PRT/100$$

$$60000 = 300000 \times 10 \times T / 100$$

$$60000 = 30000T$$

$$T = 60000/30000 = 2 \text{ years}$$

Final Answer: 2 years

8. (a) The first term of an arithmetic progression is 12 and the common difference is 10. Find the n^{th} term.

Answer:

$$a = 12, d = 10$$

$$n^{\text{th}} \text{ term} = a + (n-1)d$$

$$= 12 + (n-1) \times 10$$

$$= 12 + 10n - 10$$

$$= 10n + 2$$

Final Answer: $10n + 2$

(b) Find the amount of money accumulated at the end of 2 years after investing 500,000 shillings at a compound interest rate of 10% annually.

Answer:

$$A = P(1 + r)^t$$

$$P = 500000, r = 0.1, t = 2$$

$$A = 500000 (1 + 0.1)^2$$

$$= 500000 \times (1.1)^2$$

$$= 500000 \times 1.21$$

$$= 605000$$

Final Answer: sh 605000

9. (a) A ladder leans against a wall. If the ladder reaches 12m up the wall its foot is 9m from the base of the wall, find the length of the ladder.

Answer:

Right triangle:

$$\text{Height} = 12 \text{ m, base} = 9 \text{ m}$$

$$\text{Length} = \sqrt{(12^2 + 9^2)} = \sqrt{(144 + 81)} = \sqrt{225} = 15 \text{ m}$$

Final Answer: 15 m

(b) Given that A and B are complementary angles and $\sin A = \frac{1}{3}$, find $\sin B$. (Leave your answer as an improper fraction).

Answer:

$$A + B = 90^\circ$$

$$\sin B = \sin (90^\circ - A) = \cos A$$

$$\sin A = \frac{1}{3}$$

$$\cos A = \sqrt{(1 - \sin^2 A)} = \sqrt{(1 - (1/3)^2)} = \sqrt{(1 - 1/9)} = \sqrt{(8/9)} = \frac{2\sqrt{2}}{3}$$

Final Answer: $2\sqrt{2/3}$

10. (a) What must be added to $x^2 + 8x$ to make the expression a perfect square?

Answer:

$$x^2 + 8x$$

Complete the square: $(x + a)^2 = x^2 + 2ax + a^2$

$$2a = 8 \rightarrow a = 4$$

$$(x + 4)^2 = x^2 + 8x + 16$$

Add 16 to make it a perfect square.

Final Answer: 16

(b) Find two consecutive odd numbers whose product is 195.

Answer:

Let numbers be x and $x + 2$

$$x(x + 2) = 195$$

$$x^2 + 2x - 195 = 0$$

$$(x + 15)(x - 13) = 0$$

$$x = 13 \text{ (discard negative)}$$

Numbers: 13, 15

Final Answer: 13 and 15

11. (a) Solve by graphical method the following system of simultaneous equations:

$$4x + y = 6$$

$$5x + 2y = 9$$

Answer:

$$4x + y = 6 \rightarrow y = 6 - 4x$$

$$5x + 2y = 9 \rightarrow 2y = 9 - 5x \rightarrow y = (9 - 5x)/2$$

Points for $y = 6 - 4x$: (0, 6), (1, 2)

Points for $y = (9 - 5x)/2$: (1, 2), (0, 4.5)

Intersection at (1, 2).

Final Answer: $x = 1$, $y = 2$

(b) A farm is to be planted with sorghum and maize while observing the following constraints:

Days labour per hectare | Sorghum | Maize | Maximum total

4 | 2 | 20

Labour cost per hectare | 1400 | 1200 | 8400

Cost of fertilizer per hectare (shs) | 600 | 800 | 4800

If sorghum yields a profit of 800,000 shillings per hectare while maize yields 600,000 shillings per hectare, how many hectares should be planted with each crop for maximum profit? Carefully study

12. the frequency table shows the marks of 100 students in a Mathematics examination.

Marks | 41 – 50 | 51 – 60 | 61 – 70 | 71 – 80 | 81 – 90 | 91 – 100

Number of Students | 10 | 22 | 34 | 25 | 7 | 2

Calculate

(a) the mean in the assumed mean is 75.5,

(b) the median in two decimal places,

(c) the mode in two decimal places.

Answer:

Let x = hectares of sorghum, y = hectares of maize

Constraints (from 11b):

$$4x + 2y \leq 20 \rightarrow 2x + y \leq 10 \text{ (days)}$$

$$1400x + 1200y \leq 8400 \rightarrow 7x + 6y \leq 42 \text{ (labour cost)}$$

$$600x + 800y \leq 4800 \rightarrow 3x + 4y \leq 24 \text{ (fertilizer)}$$

$$x \geq 0, y \geq 0$$

Profit: $P = 800000x + 600000y$

Vertices:

(0, 0): $P = 0$

(0, 6): $3x + 4y = 24 \rightarrow 4y = 24 \rightarrow y = 6, 2x + y = 6 \leq 10$

$P = 600000 \times 6 = 3600000$

(6, 0): $2x + y = 10 \rightarrow 2x = 10 \rightarrow x = 5$, but $7x + 6y = 7 \times 6 = 42$

$P = 800000 \times 6 = 4800000$

(3, 4): $2x + y = 10, 3x + 4y = 24 \rightarrow 6x + 3y = 30, 6x + 8y = 48 \rightarrow 5y = 18 \rightarrow y = 3.6, x = 3.2$

Adjust: Solve correctly:

(4, 2): $7x + 6y = 42, 2x + y = 10 \rightarrow 14x + 12y = 84, 14x + 7y = 70 \rightarrow 5y = 14 \rightarrow y = 2.8, x = 3.6$

Check vertices properly:

Max at (6, 0)

Final Answer: 6 hectares sorghum, 0 hectares maize

(a) the mean in the assumed mean is 75.5,

(b) the median in two decimal places,

(c) the mode in two decimal places.

Answer:

(a) Assumed mean = 75.5

Midpoints: 45.5, 55.5, 65.5, 75.5, 85.5, 95.5

Deviations: -30, -20, -10, 0, 10, 20

Frequency $\times$ Deviation: $(-30 \times 10) + (-20 \times 22) + (-10 \times 34) + (0 \times 25) + (10 \times 7) + (20 \times 2)$

$= -300 - 440 - 340 + 0 + 70 + 40 = -970$

Mean $= 75.5 + (-970/100) = 75.5 - 9.7 = 65.8$

(b) Cumulative frequencies:

≤ 50 : 10

≤ 60 : $10 + 22 = 32$

$$\leq 70: 32 + 34 = 66$$

$$\leq 80: 66 + 25 = 91$$

$$\leq 90: 91 + 7 = 98$$

$$\leq 100: 98 + 2 = 100$$

Median (50th value) in 61–70 class:

$$\text{Median} = 60 + (50 - 32)/34 \times 10 = 60 + 5.294 = 65.29$$

(c) Mode: Modal class 61–70

$$\text{Mode} = 60 + (34 - 22)/((34 - 22) + (34 - 25)) \times 10$$

$$= 60 + 12/(12 + 9) \times 10 = 60 + (12/21) \times 10 = 60 + 5.714 = 65.71$$

Final Answer: (a) 65.8, (b) 65.29, (c) 65.71

13. (a) The figure below is a rectangular prism in which $AB = 10\text{cm}$, $BC = 8\text{cm}$ and $BF = 6\text{cm}$. Calculate the length of DP (Leave your answer in surd form).

Answer:

DP is a space diagonal in the prism.

In $\triangle BFC$: $BF = 6\text{ cm}$, $BC = 8\text{ cm}$

$$BC^2 + BF^2 = 8^2 + 6^2 = 64 + 36 = 100$$

$$DP = \sqrt{AB^2 + BC^2 + BF^2} = \sqrt{10^2 + 8^2 + 6^2} = \sqrt{100 + 64 + 36} = \sqrt{200} = 10\sqrt{2}$$

Final Answer: $10\sqrt{2}\text{ cm}$

(b) If O is the centre of a circle given below, find the value of x .

Answer:

Cyclic quadrilateral $ABCD$:

$$\angle ABC = 80^\circ, \angle ADC = x$$

Opposite angles supplementary:

$$\angle ABC + \angle ADC = 180^\circ$$

$$80^\circ + x = 180^\circ$$

$$x = 100^\circ$$

Final Answer: $x = 100^\circ$

(c) Calculate the circumference of a small circle, in kilometres, along the parallel latitude 30°N . (Leave your answer in surd form).

Answer:

Radius of Earth = 6400 km (from previous context)

Latitude = 30°N

Radius at latitude = $6400 \times \cos 30^\circ = 6400 \times \sqrt{3}/2 = 3200\sqrt{3}$ km

Circumference = $2\pi r = 2\pi \times 3200\sqrt{3} = 6400\pi\sqrt{3}$ km

Final Answer: $6400\pi\sqrt{3}$ km

14. Mr. Chapukazi commenced business on 1st June, 2011 with capital in cash 5,500,000/=

June 2 Bought goods by Cash 2,200,000/=

3 Bought furniture for Cash 600,000/=

5 Sold goods for Cash 1,700,000/=

7 Purchased goods for Cash 700,000/=

11 Purchased goods for Cash 500,000/=

13 Paid Rent for Cash 2,000,000/=

16 Cash Sales 950,000/=

20 Paid Transport for Cash 80,000/=

22 Sold goods for cash 250,000/=

26 Paid salaries for Cash 160,000/=

Record the above transactions in the cash account, balance them and extract the Trial Balance.

Answer:

Cash Account:

Date	Details	Amount	Date	Details	Amount
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Jun 1	Capital	5500000	Jun 2	Purchases	2200000
Jun 5	Sales	1700000	Jun 3	Furniture	600000
Jun 16	Sales	950000	Jun 7	Purchases	700000
Jun 22	Sales	250000	Jun 11	Purchases	500000
		Jun 13	Rent	2000000	
		Jun 20	Transport	80000	
		Jun 26	Salaries	160000	
		Jun 30	Balance c/d	1060000	
Total		8400000	Total		8400000

Trial Balance as of 30 June 2011:

Account | Debit | Credit

Capital | | 5500000

Purchases | 3400000 |

Furniture | 600000 |

Sales | | 2900000

Rent | 2000000 |

Transport | 80000 |

Salaries | 160000 |

Cash | 1060000 |

Total | 7300000 | 8400000

15. (a) Given $A = \begin{pmatrix} 2 & 6 \\ 4 & 8 \end{pmatrix}$

(4 8) find A^{-1} .

Answer:

$$A = \begin{bmatrix} 2 & 6 \\ 4 & 8 \end{bmatrix}$$

$$|A| = (2 \times 8) - (6 \times 4) = 16 - 24 = -8$$

$$\text{adj}(A) = \begin{bmatrix} 8 & -6 \\ -4 & 2 \end{bmatrix}$$

$$A^{-1} = (1/|A|) \times \text{adj}(A) = (1/-8) \times \begin{bmatrix} 8 & -6 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 3/4 \\ 1/2 & -1/4 \end{bmatrix}$$

$$\text{Final Answer: } \begin{bmatrix} -1 & 3/4 \\ 1/2 & -1/4 \end{bmatrix}$$

(b) Use the results obtained in part (a) to find the point of intersection of the following system of simultaneous equations:

$$2x + 6y = 22$$

$$4x + 8y = 32$$

Answer:

$$\begin{bmatrix} 2 & 6 \\ 4 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 22 \\ 32 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = A^{-1} \begin{bmatrix} 22 \\ 32 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -1 & 3/4 \\ 1/2 & -1/4 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 & 3/4 \\ 1/2 & -1/4 \end{bmatrix} \begin{bmatrix} 22 \\ 32 \end{bmatrix}$$

$$x = (-1) \times 22 + (3/4) \times 32 = -22 + 24 = 2$$

$$y = (1/2) \times 22 + (-1/4) \times 32 = 11 - 8 = 3$$

$$\text{Final Answer: } (2, 3)$$

(c) Find the image of the point obtained in part (b) above under a rotation 90° anticlockwise.

Answer:

$$\text{Point} = (2, 3)$$

$$\text{Rotation } 90^\circ \text{ anticlockwise: } (x, y) \rightarrow (-y, x)$$

$$(2, 3) \rightarrow (-3, 2)$$

$$\text{Final Answer: } (-3, 2)$$

16. (a) A die and a coin are tossed together. What is the probability of getting a tail and an even number?

Answer:

Coin: $P(\text{tail}) = 1/2$

Die: Even numbers (2, 4, 6), $P(\text{even}) = 3/6 = 1/2$

$P(\text{tail and even}) = (1/2) \times (1/2) = 1/4$

Final Answer: $1/4$

(b) Draw the graph of the inverse of $R = \{(x, y) : y \geq 0 \text{ and } y \leq x\}$. Find its domain and range.

Answer:

$R = \{(x, y) : y \geq 0, y \leq x\}$

Inverse R^{-1} : Swap x and $y \rightarrow \{(x, y) : x \geq 0, x \leq y\}$

Graph: Region above $y = x$, $x \geq 0$

Domain of R^{-1} : $x \geq 0$

Range of R^{-1} : $y \geq 0$

Final Answer: Domain: $[0, \infty)$, Range: $[0, \infty)$

(c) Without using a table of values, draw the graph of $y = x^2 - 4x + 2$ and use it to solve the equation $x^2 - 4x + 5$.

Answer:

$y = x^2 - 4x + 2$

Complete the square: $y = (x - 2)^2 - 4 + 2 = (x - 2)^2 - 2$

Vertex: (2, -2)

y-intercept: (0, 2)

x-intercepts: $x^2 - 4x + 2 = 0 \rightarrow x = [4 \pm \sqrt{(16 - 8)}]/2 = [4 \pm 2\sqrt{2}]/2 = 2 \pm \sqrt{2}$

Solve $x^2 - 4x + 5 = 0 \rightarrow y = x^2 - 4x + 2 = -3$

$(x - 2)^2 - 2 = -3$

$(x - 2)^2 = -1$ (no real solutions)