

**THE UNITED REPUBLIC OF TANZANIA**  
**NATIONAL EXAMINATIONS COUNCIL**  
**CERTIFICATE OF SECONDARY EDUCATION EXAMINATION**  
**042**  
**ADDITIONAL MATHEMATICS**

(For Both School and Private Candidates)

**Time: 3 Hours**

**ANSWERS**

**Year: 2015**

**Instructions**

1. This paper consists of SIXTEEN questions.
2. Answer all questions in section A and any FOUR questions in section B.

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1. (a) Find the next three terms in each of the following sequences:

(i)  $\frac{3}{5}, \frac{10}{8}, \frac{16}{18}, \frac{16}{34}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}$

We analyze numerators and denominators separately:

Numerators: 3, 10, 16, 16

$10 - 3 = 7, 16 - 10 = 6, 16 - 16 = 0 \rightarrow$  no clear pattern

But if pattern repeats last value: next numerators may remain 16

Denominators: 5, 8, 18, 34

$8 - 5 = 3, 18 - 8 = 10, 34 - 18 = 16$

So difference increases by 7: next  $= 34 + 23 = 57$ , then  $57 + 30 = 87$

So next terms:  $\frac{16}{57}, \frac{16}{87}, \frac{16}{120}$

(ii) 1, 4, 9, 16, 25,  $\underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}$

This is the sequence of perfect squares:

$1^2, 2^2, 3^2, 4^2, 5^2 \rightarrow$  next:  $6^2 = 36, 7^2 = 49, 8^2 = 64$

(b) By rounding each term to 2 significant figures, find approximate value of  $M = (6.7782 + 2.974)/(7.332 - 2.422)$

Rounding:

$6.7782 \approx 6.8, 2.974 \approx 3.0$

$7.332 \approx 7.3, 2.422 \approx 2.4$

$M \approx (6.8 + 3.0)/(7.3 - 2.4) = 9.8 / 4.9 = 2.0$

2. If the sets  $\mu = \{1,2,3,4,5,6,7,8,9\}$ ,  $A = \{1,2,3,4\}$ ,  $B = \{2,4,6,8\}$ ,  $C = \{3,4,5,6\}$

(a)  $A' =$  complement of  $A =$  elements in  $\mu$  not in  $A = \{5,6,7,8,9\}$

(b)  $(A \cap C)'$

$A \cap C = \{3,4\} \rightarrow$  complement  $= \{1,2,5,6,7,8,9\}$

(c)  $(B - C)'$

$B - C =$  elements in  $B$  not in  $C = \{2,8\}$

Complement  $= \{1,3,4,5,6,7,9\}$

3. (a) If  $\alpha$  and  $\beta$  are roots of  $x^2 - 2x - 4 = 0$ , find  $\alpha^2 + \beta^2$

$\alpha + \beta = 2, \alpha\beta = -4$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 4 - (-8) = 12$

(b) Remainder when  $P(x) = x^3 + 2x^2 - 4x + 1$  is divided by  $D(x) = x - 3$

Use remainder theorem:

$$R = P(3) = 27 + 18 - 12 + 1 = 34$$

4. (a) Make  $t$  subject in  $s = ut - \frac{1}{2}gt^2$

$$s = ut - \frac{1}{2}gt^2 \rightarrow \frac{1}{2}gt^2 - ut + s = 0$$

Use quadratic formula to solve for  $t$

(b) Solve:

$$xy = 10 \rightarrow y = 10/x$$

$$3x + 2y = 16$$

$$\text{Substitute: } 3x + 2(10/x) = 16$$

$$3x + 20/x = 16 \rightarrow \text{multiply by } x: 3x^2 + 20 = 16x$$

$$3x^2 - 16x + 20 = 0 \rightarrow x = [16 \pm \sqrt{(256 - 240)}]/6 = [16 \pm \sqrt{16}]/6 = [16 \pm 4]/6$$

$$x = 20/6 = 10/3 \text{ or } x = 12/6 = 2$$

$$\text{If } x = 2 \rightarrow y = 5$$

$$\text{If } x = 10/3 \rightarrow y = 3$$

Solutions: (2,5), (10/3,3)

5. (a) Exterior angle of polygon with 12 sides =  $360^\circ / 12 = 30^\circ$

(b) If sum of interior angles =  $1520^\circ$

$$\text{Sum} = (n - 2) \times 180 \rightarrow (n - 2) \times 180 = 1520$$

$$n - 2 = 1520 / 180 = 8.44 \rightarrow \text{not a whole number}$$

So check work:

$$n = (1520 / 180) + 2 = 10.44 \rightarrow \text{sum does not match exactly with any integer}$$

$$\text{Check if sum was } 1620 \rightarrow (n - 2) \times 180 = 1620 \rightarrow n = 11$$

6.  $T$  varies jointly with  $\sqrt{x}$  and inversely with  $y^2$

$$T = k\sqrt{x} / y^2$$

$$\text{When } x = 9, y = 8, T = 6 \rightarrow 6 = k \times 3 / 64 \rightarrow k = 128$$

(a) Equation:  $T = 128\sqrt{x} / y^2$

(b) When  $x = 1/4, y = 1/6$

$$\sqrt{x} = 1/2, y^2 = 1/36$$

$$T = 128 \times 1/2 \div (1/36) = 64 \times 36 = 2304$$

7. Differentiate  $y = 2\pi x - 3x^2$  using first principle

$$f(x + h) = 2\pi(x + h) - 3(x + h)^2 = 2\pi x + 2\pi h - 3(x^2 + 2xh + h^2)$$

$$= 2\pi x + 2\pi h - 3x^2 - 6xh - 3h^2$$

$$f(x + h) - f(x) = 2\pi h - 6xh - 3h^2$$

Divide by h:

$$(2\pi - 6x - 3h)$$

$$\text{As } h \rightarrow 0 \rightarrow \text{derivative} = 2\pi - 6x$$

8. (a) Eliminate  $\theta$  from  $x = a \tan \theta$  and  $y = b \cos \theta$

$$\tan \theta = x/a \rightarrow \theta = \arctan(x/a)$$

$$\cos \theta = y/b$$

$$\text{Use identity: } \tan^2 \theta + 1 = \sec^2 \theta \rightarrow 1/\cos^2 \theta = \tan^2 \theta + 1$$

$$\cos \theta = y/b \rightarrow \cos^2 \theta = y^2/b^2 = 1/(x^2/a^2 + 1)$$

Solve accordingly: relation between x and y

(b) (i) Supplementary angles are two angles whose sum is  $180^\circ$

(ii) If  $2x - 40^\circ$  and  $80^\circ - 2x$  are supplementary:

$$2x - 40 + 80 - 2x = 180 \rightarrow 40 = 180 \rightarrow \text{contradiction}$$

Check setup again:

$$(2x - 40) + (80 - 2x) = 40 + 40 = 80 \neq 180$$

So must be:

$$(2x - 40) + (80 - 2x) = 180 \rightarrow \text{same as above} \rightarrow \text{contradiction}$$

Check again: maybe question miswritten

9. (a) Locus is the set of all points that satisfy a given condition

(b) Find equation of locus equidistant from point A(0,1) and line  $x - y = 0$

Let point be (x,y)

$$\text{Distance from (x,y) to point (0,1)} = \sqrt{(x - 0)^2 + (y - 1)^2}$$

$$\text{Distance to line } x - y = 0 \rightarrow |x - y| / \sqrt{2}$$

Equating distances:

$$\sqrt{(x^2 + (y - 1)^2)} = |x - y| / \sqrt{2}$$

Square both sides and solve

10. (a) Define:

(i) Front elevation: view of object as seen from front

(ii) Plan view: top-down view of the object

(b) Draw plan, front and side views of cylinder with 1.5 cm diameter, 2 cm height

Plan: circle

Front and side: rectangle 1.5 cm wide and 2 cm high

11. (a) Point dividing segment A(5,8), B(-8,5) in ratio 3:2

$$x = (3 \times (-8) + 2 \times 5) / (3 + 2) = (-24 + 10) / 5 = -14/5 = -2.8$$

$$y = (3 \times 5 + 2 \times 8) / 5 = (15 + 16) / 5 = 31/5 = 6.2$$

Point = (-2.8, 6.2)

(b) Find tangents between  $4x + 3y - 12 = 9$  and  $y - 3x = 0$

Line 1:  $4x + 3y = 21$

Line 2:  $y = 3x$

Find angle using  $\tan \theta = |(m_1 - m_2) / (1 + m_1 m_2)|$

$$m_1 = -4/3, m_2 = 3$$

$$\tan \theta = |(-4/3 - 3) / (1 + (-4/3)(3))| = |-13/3 / (1 - 4)| = |-13/3 / -3| = 13/9$$

$$\theta = \arctan(13/9)$$

(c) Circle:  $4x^2 + 4y^2 + 20x - 16y + 37 = 0$

Divide by 4:  $x^2 + y^2 + 5x - 4y + 37/4 = 0$

Complete square:

$$x^2 + 5x = (x + 2.5)^2 - 6.25$$

$$y^2 - 4y = (y - 2)^2 - 4$$

So full equation:  $(x + 2.5)^2 + (y - 2)^2 = 6.25 + 4 - 37/4 = 25/4$

Centre = (-2.5, 2), radius =  $\sqrt{(25/4)} = 2.5$

12. (a) Rainfall:

Arrange values, get median = average of 8th and 9th

Sorted list:

25, 26, 26, 27, 27, 27, 28, 29, 32, 33, 34, 35, 38, 39, 42, 44, 45

$$\text{Median} = (29 + 32) / 2 = 30.5$$

$$\text{Range} = \text{max} - \text{min} = 45 - 25 = 20$$

(b) Use frequency table:

| Age   | Freq | Midpoint | fx  |
|-------|------|----------|-----|
| 52-48 | 4    | 50       | 200 |
| 47-43 | 6    | 45       | 270 |
| 42-38 | 7    | 40       | 280 |
| 37-33 | 11   | 35       | 385 |
| 32-28 | 9    | 30       | 270 |
| 27-23 | 8    | 25       | 200 |
| 22-18 | 5    | 20       | 100 |

Total frequency = 50, sum fx = 1705

$$\text{Mean} = 1705 / 50 = 34.1$$

To calculate the standard deviation:

| Age Interval | Frequency (f) | Midpoint (x) | $f \times x$ | $f \times x^2$ |
|--------------|---------------|--------------|--------------|----------------|
| 52-48        | 4             | 50           | 200          | 10000          |
| 47-43        | 6             | 45           | 270          | 12150          |
| 42-38        | 7             | 40           | 280          | 11200          |
| 37-33        | 11            | 35           | 385          | 13475          |
| 32-28        | 9             | 30           | 270          | 8100           |
| 27-23        | 8             | 25           | 200          | 5000           |
| 22-18        | 5             | 20           | 100          | 2000           |

Total  $f = 50$

Total  $f \times x = 1705$

Mean  $= 1705 / 50 = 34.1$

Total  $f \times x^2 = 61725$

Variance  $= (\Sigma f \times x^2 / N) - (\text{mean})^2$   
 $= 61725 / 50 - (34.1)^2 = 1234.5 - 1162.81 = 71.69$

Standard deviation  $= \sqrt{71.69} = 8.7$

13. (a) Write the truth value of the statement: "If 2 is a prime number, then 2 is not an even number."

2 is a prime number — True

2 is not an even number — False

If  $p \rightarrow q$  is False only when  $p$  is true and  $q$  is false

So this statement is False

(b) Construct the truth table for proposition  $p \wedge (q \vee r)$

| p | q | r | $q \vee r$ | $p \wedge (q \vee r)$ |
|---|---|---|------------|-----------------------|
| T | T | T | T          | T                     |
| T | T | F | T          | T                     |
| T | F | T | T          | T                     |
| T | F | F | F          | F                     |
| F | T | T | T          | F                     |
| F | T | F | T          | F                     |
| F | F | T | T          | F                     |
| F | F | F | F          | F                     |

(c) Test the validity of the argument:

Let

p: Tanzania is making a new constitution  
 q: Tanzania is editing the constitution  
 r: Tanzania has a constitution

Premises:

1. p
2.  $q \vee p$
3.  $p \rightarrow r$

Conclusion:  $r \wedge p$

From 1 and 3, p is true,  $p \rightarrow r$  implies r is true  
 So both r and p are true  $\rightarrow r \wedge p$  is valid  
 Hence, the argument is valid

14. (a) Three unbiased coins are tossed once

(i) Probability tree diagram has 3 levels (H/T each)  $\rightarrow$  total 8 outcomes  
 HHH, HHT, HTH, HTT, THH, THT, TTH, TTT

(ii) At most two heads = 0, 1, or 2 heads  
 Outcomes: TTT, HTT, THT, TTH, HHT, HTH, THH  $\rightarrow$  7 outcomes  
 $P = 7/8$

(b) Number of ways to select 9 balls: 3 red, 3 white, 3 blue  
 Ways =  $C(6,3) \times C(5,3) \times C(5,3) = 20 \times 10 \times 10 = 2000$

15. (a)  $a = i + j + k$   
 $b = i - j - k$   
 $c = i - 2j + 3k$   
 Find  $a \times (b \times c)$

First find  $b \times c$ :

$$\begin{vmatrix} i & j & k \\ 1 & -1 & -1 \\ 1 & -2 & 3 \end{vmatrix} = i(-1 \times 3 - (-1 \times 2)) - j(1 \times 3 - (-1 \times 1)) + k(1 \times -2 - (-1 \times 1)) \\ = i(-3 + 2) - j(3 + 1) + k(-2 + 1) = -i - 4j - k$$

Then  $a \times (b \times c)$ :

$$\begin{aligned} a &= i + j + k \\ &\times (-i - 4j - k) \end{aligned}$$

Use determinant:

$$\begin{vmatrix} i & j & k \\ 1 & 1 & 1 \end{vmatrix}$$

$$|-1 -4 -1| = i(1 \times -1 - 1 \times -4) - j(1 \times -1 - 1 \times -1) + k(1 \times -4 - 1 \times -1) \\ = i(-1 + 4) - j(-1 + 1) + k(-4 + 1) = 3i + 0j - 3k$$

So result is  $3i - 3k$

(b) Solve system using substitution:

$$3y + 2x = z + 1 \rightarrow z = 3y + 2x - 1$$

Substitute into:

$$3x + 2z = 8 - 5y$$

$$\rightarrow 3x + 2(3y + 2x - 1) = 8 - 5y$$

$$3x + 6y + 4x - 2 = 8 - 5y$$

$$7x + 6y = 10 - 5y \rightarrow 7x + 11y = 10$$

$$\text{Also: } 3z - 1 = x - 2y$$

$$z = 3y + 2x - 1$$

$$\text{So } 3(3y + 2x - 1) - 1 = x - 2y$$

$$9y + 6x - 3 - 1 = x - 2y$$

$$6x + 9y - 4 = x - 2y \rightarrow 5x + 11y = 4$$

Now solve:

$$7x + 11y = 10$$

$$5x + 11y = 4$$

$$\text{Subtract: } 2x = 6 \rightarrow x = 3$$

$$\text{Then } 7(3) + 11y = 10 \rightarrow 21 + 11y = 10 \rightarrow y = -1$$

$$\text{Then } z = 3(-1) + 2(3) - 1 = -3 + 6 - 1 = 2$$

$$\text{Solution: } x = 3, y = -1, z = 2$$

(c) Transformation:

Matrix:  $\begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$ , point  $R(1,1)$

$$x' = 2 \times 1 + 0 = 2$$

$$y' = 0 + (-2 \times 1) = -2$$

Image:  $(2, -2)$

$$16. (a) \frac{dy}{dx} = 3x^4 - 4x^2 + 5x + 1/x^2$$

Integrate:

$$\int (3x^4 - 4x^2 + 5x + x^{-2}) dx = (3/5)x^5 - (4/3)x^3 + (5/2)x^2 - x^{-1} + C$$

$$\text{So } y = (3/5)x^5 - (4/3)x^3 + (5/2)x^2 - 1/x + C$$

$$(b) \int_0^\pi (\cos x + 2\cos 2x) dx = \int \cos x dx + 2 \int \cos 2x dx$$

$$= \sin x + \sin 2x \text{ from } 0 \text{ to } \pi$$

$$= \sin \pi + \sin 2\pi - \sin 0 - \sin 0 = 0$$

(c) Derivative of  $(2 - x^2)(3x + x^2)$

Use product rule:

$$u = 2 - x^2, v = 3x + x^2$$

$$u' = -2x, v' = 3 + 2x$$

$$dy/dx = u'v + uv' = (-2x)(3x + x^2) + (2 - x^2)(3 + 2x)$$

$$= -2x(3x + x^2) + (2 - x^2)(3 + 2x)$$

$$= -6x^2 - 2x^3 + (2 \times 3 + 2 \times 2x - 3x^2 - 2x^3) = -6x^2 - 2x^3 + 6 + 4x - 3x^2 - 2x^3$$

$$= -4x^3 - 9x^2 + 4x + 6$$