

THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL
CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

041

BASIC MATHEMATICS

(For Both School and Private Candidates)

Time: 3 Hours

ANSWERS

Year: 2016

Instructions

1. This paper consists of Section A and B.
2. Answer all questions in section A and any four questions in section B.

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1. (a) From the set of numbers {1,3,4,5,6,8,10,15,17,21,27}, write down:

(i) the prime numbers,

(ii) the multiples of 3,

(iii) the factors of 60.

(b) Four wooden rods with lengths of 70 cm, 119 cm, 84 cm and 105 cm are cut into pieces of the same length. Find the greatest possible length for these pieces if no wood is left over.

(a)(i) Prime numbers: Check each number for divisibility only by 1 and itself.

1: Not prime

3, 5, 17: Prime

4: Divisible by 2

6: Divisible by 2, 3

8: Divisible by 2, 4

10: Divisible by 2, 5

15: Divisible by 3, 5

21: Divisible by 3, 7

27: Divisible by 3, 9

Answer: {3, 5, 17}

(a)(ii) Multiples of 3: Numbers divisible by 3.

3, 6, 15, 21, 27

Answer: {3, 6, 15, 21, 27}

(a)(iii) Factors of 60: Numbers in set that divide 60.

$60 = 1 \times 60, 2 \times 30, 3 \times 20, 4 \times 15, 5 \times 12, 6 \times 10$

From set: 1, 3, 4, 5, 6, 10, 15

Answer: {1, 3, 4, 5, 6, 10, 15}

(b) Greatest possible length = GCD of 70, 119, 84, 105

Factorize:

$$70 = 2 \times 5 \times 7$$

$$119 = 7 \times 17$$

$$84 = 2^2 \times 3 \times 7$$

$$105 = 3 \times 5 \times 7$$

$$\text{GCD} = 7$$

Answer: 7 cm

2. (a) Solve for x in the equation $9^{(x-3)} \times 81^{(1-x)} = 27^{-x}$.

(b) Show that $(\log 16 + \log 81) / (\log 27 + \log 8) = 4/3$.

$$(a) 9 = 3^2, 81 = 3^4, 27 = 3^3$$

$$9^{(x-3)} = (3^2)^{(x-3)} = 3^{(2x-6)}$$

$$81^{(1-x)} = (3^4)^{(1-x)} = 3^{(4-4x)}$$

$$27^{-x} = (3^3)^{-x} = 3^{-3x}$$

$$\text{Left: } 3^{(2x-6)} \times 3^{(4-4x)} = 3^{(2x-6+4-4x)} = 3^{(4-2x-6)}$$

$$\text{Equation: } 3^{(4-2x-6)} = 3^{-3x}$$

$$\text{Exponents: } 4 - 2x - 6 = -3x$$

$$-2x - 2 = -3x$$

$$x = -2$$

Answer: $x = -2$

$$(b) \text{ Left: } \log 16 + \log 81 = \log(16 \times 81)$$

$$16 = 2^4, 81 = 3^4$$

$$\log(16 \times 81) = \log(2^4 \times 3^4) = \log(2^4 \times 3^4) = 4(\log 2 + \log 3)$$

$$\text{Right: } \log 27 + \log 8 = \log(27 \times 8)$$

$$27 = 3^3, 8 = 2^3$$

$$\log(27 \times 8) = \log(3^3 \times 2^3) = 3(\log 3 + \log 2)$$

$$\text{Ratio: } [4(\log 2 + \log 3)] / [3(\log 3 + \log 2)] = 4/3$$

3. (a) By substituting $a = 1/x$ and $b = 1/y$ in the system of equations: $\{-1/x + 6/(2y) = -1, 3/x - 2/y = 5\}$, find the solution set (x, y) .

(b) Let U be a universal set and A and B be the subsets of U where: $U = \{1,2,3,4,5,6,7,8,9,10\}$, $A = \{\text{odd numbers}\}$ and $B = \{\text{prime numbers}\}$

(i) Represent this information in a Venn diagram.

(ii) Find $A \cap B'$ and $(A \cup B)'$.

(a) Substitute $a = 1/x$, $b = 1/y$

Equations become:

$$-a + 3b = -1 \quad (1)$$

$$3a - 2b = 5 \quad (2)$$

$$\text{Multiply (1) by 2: } -2a + 6b = -2 \quad (3)$$

$$\text{Multiply (2) by 3: } 9a - 6b = 15 \quad (4)$$

Add (3) and (4):

$$7a = 13$$

$$a = 13/7$$

Substitute $a = 13/7$ in (1):

$$-(13/7) + 3b = -1$$

$$3b = -1 + 13/7 = 6/7$$

$$b = 2/7$$

$$x = 1/a = 7/13, y = 1/b = 7/2$$

Answer: $(7/13, 7/2)$

(b)(i) $U = \{1,2,3,4,5,6,7,8,9,10\}$

$A = \{1,3,5,7,9\}$, $B = \{2,3,5,7\}$

Venn diagram

Circle A: 1, 3, 5, 7, 9

Circle B: 2, 3, 5, 7

Overlap: 3, 5, 7

Outside: 4, 6, 8, 10

$$(b)(ii) B' = \{1, 4, 6, 8, 9, 10\}$$

$$A \cap B' = \{1, 3, 5, 7, 9\} \cap \{1, 4, 6, 8, 9, 10\} = \{1, 9\}$$

$$A \cup B = \{1, 2, 3, 5, 7, 9\} \cup \{2, 3, 5, 7\} = \{1, 2, 3, 5, 7, 9\}$$

$$(A \cup B)' = \{4, 6, 8, 10\}$$

$$\text{Answer: } A \cap B' = \{1, 9\}, (A \cup B)' = \{4, 6, 8, 10\}$$

4. (a) Given vectors $a = 6i + 12j$, $b = 17i + 18j$:

(i) Find the vector $c = 2a - b$ and its magnitude correctly to 3 significant figures.

(ii) Represent vector c in part (a)(i) on the x-y plane.

(b) Find the equation of the line passing through the midpoint of the points $A(-3, 2)$ and $B(1, -4)$ and which is perpendicular to line AB .

$$(a)(i) c = 2a - b$$

$$2a = 2(6i + 12j) = 12i + 24j$$

$$b = 17i + 18j$$

$$c = (12i + 24j) - (17i + 18j) = (12 - 17)i + (24 - 18)j = -5i + 6j$$

$$\text{Magnitude: } |c| = \sqrt{(-5)^2 + 6^2} = \sqrt{25 + 36} = \sqrt{61} \approx 7.81$$

$$\text{Answer: } c = -5i + 6j, |c| = 7.81$$

(a)(ii) [Cannot draw; describe]

Vector $c = -5i + 6j$: Arrow from $(0, 0)$ to $(-5, 6)$ on x-y plane.

(b) Midpoint of $A(-3, 2)$ and $B(1, -4)$:

$$x = (-3 + 1)/2 = -1, y = (2 - 4)/2 = -1$$

Midpoint: $(-1, -1)$

$$\text{Slope of } AB: (2 - (-4)) / (-3 - 1) = 6 / -4 = -3/2$$

Perpendicular slope: $\frac{2}{3}$

Line through $(-1, -1)$:

$$y - (-1) = \left(\frac{2}{3}\right)(x - (-1))$$

$$y + 1 = \left(\frac{2}{3}\right)(x + 1)$$

$$y = \left(\frac{2}{3}\right)x + \frac{2}{3} - 1 = \left(\frac{2}{3}\right)x - \frac{1}{3}$$

Answer: $y = \left(\frac{2}{3}\right)x - \frac{1}{3}$

5. (a) In triangle ABC, X, Y and Z are the midpoints of sides AB, AC and BC respectively. If $ZX = ZY$ and $\angle XZB = \angle YZC = 90^\circ$;

(i) Represent this information diagrammatically,

(ii) Show that $AB = AC$.

(b) The areas of two similar polygons are 27 and 48 square metres. If the length of one side of the smaller polygon is 4.5 cm, find the length of the corresponding side of the larger polygon.

(a)(i)

Triangle ABC, X midpoint of AB, Y midpoint of AC, Z midpoint of BC

Line $ZX \perp AB$ at X, $ZY \perp AC$ at Y

$$ZX = ZY$$

(a)(ii) In triangles ABZ and ACZ:

$$AZ = AZ \text{ (common)}$$

$$\angle XZB = \angle YZC = 90^\circ$$

$$ZX = ZY \text{ (given)}$$

By RHS congruence, triangle ABZ $\cong$ triangle ACZ

Answer: $AB = AC$ by RHS

$$(b) \text{ Area ratio} = \frac{27}{48} = \frac{9}{16}$$

$$\text{Linear ratio} = \sqrt{\frac{9}{16}} = \frac{3}{4}$$

$$\text{Smaller side} = 4.5 \text{ cm}$$

Larger side = $4.5 \times (4/3) = 6$ cm

Answer: 6 cm

6. (a) The number of tablets given to a patient was found to be directly proportional to the weight of the patient. If a patient with 36 kg was given 9 tablets, find how many tablets would be given to a patient whose weight is 48 kg.

(b) Four people can eat 2 bags of rice each weighing 10 kg in 12 days. How many people can eat 6 bags of rice of the same weight in 18 days?

(a) Tablets $\propto$ weight

$$T = kW$$

$$9 = k \times 36$$

$$k = 9/36 = 1/4$$

$$T = (1/4) \times 48 = 12$$

Answer: 12 tablets

(b) Total rice = $2 \times 10 = 20$ kg for 4 people in 12 days

$$\text{Rate per person} = 20 / (4 \times 12) = 20/48 = 5/12 \text{ kg/day}$$

$$\text{New total rice} = 6 \times 10 = 60 \text{ kg in 18 days}$$

$$\text{Total consumption needed} = 60 / 18 = 10/3 \text{ kg/day}$$

$$\text{Number of people} = (10/3) / (5/12) = 10/3 \times 12/5 = 8$$

Answer: 8 people

7. (a) Mariam, Selina and Moses contributed 800,000, 1,200,000 and 850,000 shillings respectively while starting their business.

(i) Find the ratio of their contributions in simplest form.

(ii) If the business made a profit of 1,900,000 shillings; find how much each got if the profit was shared in the same ratio as their contributions.

(b) A dealer bought 10 books for 200,000. He sold $2/5$ of them at 30,000 shillings each and the remaining at 25,000 shillings each. What was his percentage profit?

(a)(i) Contributions: 800,000 : 1,200,000 : 850,000

Simplify: 800 : 1200 : 850

Divide by 50: 16 : 24 : 17

Answer: 16:24:17

(a)(ii) Total parts = 16 + 24 + 17 = 57

Profit per part = 1,900,000 / 57 $\approx$ 33,333.33

Mariam: 16 $\times$ 33,333.33 $\approx$ 533,333.28

Selina: 24 $\times$ 33,333.33 $\approx$ 799,999.92

Moses: 17 $\times$ 33,333.33 $\approx$ 566,666.61

Answer: 533,333, 800,000, 566,667

(b) Cost price = 200,000

Books sold: $\frac{2}{5} \times 10 = 4$ at 30,000 each = $4 \times 30,000 = 120,000$

Remaining: 6 at 25,000 each = $6 \times 25,000 = 150,000$

Total sales = 120,000 + 150,000 = 270,000

Profit = 270,000 - 200,000 = 70,000

% profit = $(70,000 / 200,000) \times 100 = 35\%$

Answer: 35%

8. (a) The 8th term of an arithmetic progression is 9 greater than the 5th term and the 10th term is 10 times the 2nd term. Find the common difference and the first term of the arithmetic progression.

(b) The sum of the first two terms of a geometric progression is 18 whereas the sum of the second and third term is 54, find the first term and the common ratio.

(a) AP: $a_n = a + (n-1)d$

8th term: $a + 7d$

5th term: $a + 4d$

$a + 7d = (a + 4d) + 9$

$$3d = 9$$

$$d = 3$$

$$10\text{th term: } a + 9d$$

$$2\text{nd term: } a + d$$

$$a + 9d = 10(a + d)$$

$$a + 9(3) = 10(a + 3)$$

$$a + 27 = 10a + 30$$

$$-3 = 9a$$

$$a = -1/3$$

$$\text{Answer: } d = 3, a = -1/3$$

$$(b) \text{ GP: } a, ar, ar^2$$

$$a + ar = 18 \quad (1)$$

$$ar + ar^2 = 54 \quad (2)$$

$$\text{From (2): } ar(1 + r) = 54$$

$$\text{From (1): } a(1 + r) = 18$$

$$\text{Divide: } (ar(1 + r)) / (a(1 + r)) = 54/18$$

$$r = 3$$

$$\text{Substitute } r = 3 \text{ in (1):}$$

$$a + 3a = 18$$

$$4a = 18$$

$$a = 4.5$$

$$\text{Answer: } a = 4.5, r = 3$$

9. (a) A river with parallel banks is 20 m wide. If P and Q are two points on either side of the river, as shown in the figure below, find the distance PQ.

(b) In the triangle LMN, LM = 5 m, LN = 6 m and angle MLN = 66°. Find MN.

(a)

Assume PQ perpendicular to banks, distance = river width

Answer: 20 m

(b) In triangle LMN, use cosine rule:

$$MN^2 = LM^2 + LN^2 - 2(LM)(LN)\cos(\angle MLN)$$

$$MN^2 = 5^2 + 6^2 - 2(5)(6)\cos(66^\circ)$$

$$\cos(66^\circ) \approx 0.4067$$

$$MN^2 = 25 + 36 - 60(0.4067) \approx 61 - 24.402 = 36.598$$

$$MN = \sqrt{36.598} \approx 6.05 \text{ m}$$

Answer: 6.05 m

10. (a) If one of the roots of the quadratic equation $x^2 + bx + 24 = 0$ is $1\frac{1}{2}$, find the value of b.

(b) Two numbers differ by 3. If the sum of their reciprocals is $\frac{7}{10}$, find the numbers.

(a) Root $x = \frac{3}{2}$

$$\text{Substitute: } (\frac{3}{2})^2 + b(\frac{3}{2}) + 24 = 0$$

$$\frac{9}{4} + (\frac{3b}{2}) + 24 = 0$$

$$\frac{9}{4} + \frac{3b}{2} + \frac{96}{4} = 0$$

$$\frac{3b}{2} + \frac{105}{4} = 0$$

$$\frac{3b}{2} = -\frac{105}{4}$$

$$b = -\frac{105}{4} \times \frac{2}{3} = -\frac{35}{2}$$

Answer: $b = -\frac{35}{2}$

(b) Let numbers be x, x+3

$$\frac{1}{x} + \frac{1}{(x+3)} = \frac{7}{10}$$

$$\frac{(x+3 + x)}{(x(x+3))} = \frac{7}{10}$$

$$2x + 3 = (\frac{7}{10})x(x+3)$$

$$20x + 30 = 7x^2 + 21x$$

$$7x^2 + x - 30 = 0$$

$$x = [-1 \pm \sqrt{1 + 840}] / 14 = [-1 \pm 29] / 14$$

$$x = 2 \text{ or } x = -30/7$$

Numbers: (2, 5) or (-30/7, -9/7)

Answer: 2, 5 or -30/7, -9/7

11. A shopkeeper sells refrigerators and washing machines. Each refrigerator takes up 1.8 m² of space and costs 300,000 shillings; whereas each washing machine takes up 1.5 m² of space and costs 500,000 shillings. The owner of the shop has 6,000,000 shillings to spend and has 27 m² of space.

(a) Write down all the inequalities which represent the given information.

(b) If he makes a profit of 30,000 shillings on each refrigerator and 40,000 shillings on each washing machine, find how many refrigerators and washing machines he should sell for maximum profit.

(a) Let x = refrigerators, y = washing machines

$$\text{Space: } 1.8x + 1.5y \leq 27$$

$$\text{Cost: } 300,000x + 500,000y \leq 6,000,000$$

$$x \geq 0, y \geq 0$$

$$\text{Answer: } 1.8x + 1.5y \leq 27, 300,000x + 500,000y \leq 6,000,000, x \geq 0, y \geq 0$$

(b) Profit: $P = 30,000x + 40,000y$

Simplify constraints:

$$1.8x + 1.5y \leq 27 \rightarrow 6x + 5y \leq 90$$

$$300,000x + 500,000y \leq 6,000,000 \rightarrow 3x + 5y \leq 60$$

Vertices:

$$(0,0): P = 0$$

$$(0,12): 6x + 5y = 90, y = 12; 3x + 5(12) \leq 60, \text{ valid}$$

$$P = 30,000(0) + 40,000(12) = 480,000$$

$$(15,0): 6x = 90, x = 15; 3(15) \leq 60, \text{ valid}$$

$$P = 30,000(15) + 40,000(0) = 450,000$$

$$\text{Intersection: } 6x + 5y = 90, 3x + 5y = 60$$

Subtract: $3x = 30$

$$x = 10, y = (90 - 6(10))/5 = 6$$

$$(10,6): P = 30,000(10) + 40,000(6) = 300,000 + 240,000 = 540,000$$

Maximum at (10,6)

Answer: 10 refrigerators, 6 washing machines

12. The following were the scores of 35 students in a mathematics mock examination:

07,19,78,53,43,67,12,54,27,22,33,80,25,58,50,36,65,33,16,19,34,20,55,27,37,41,04,32,48,28,70,31,61,08,
35

(a) Prepare the frequency distribution table using the class intervals: 0-9, 10-19, 20-29, etc.

(b) Which class interval has more students?

(c) Represent the information in a histogram and a frequency polygon and then find the mode.

(d) Calculate the median mark.

(a) Frequency distribution:

0-9: 04, 07, 08 $\rightarrow$ 3

10-19: 12, 16, 19, 19 $\rightarrow$ 4

20-29: 20, 22, 25, 27, 27, 28 $\rightarrow$ 6

30-39: 31, 32, 33, 33, 34, 35, 36, 37 $\rightarrow$ 8

40-49: 41, 43, 48 $\rightarrow$ 3

50-59: 50, 53, 54, 55, 58 $\rightarrow$ 5

60-69: 61, 65, 67 $\rightarrow$ 3

70-79: 70, 78 $\rightarrow$ 2

80-89: 80 $\rightarrow$ 1

Table:

Class	Frequency
0-9	3

10-19	4
20-29	6
30-39	8
40-49	3
50-59	5
60-69	3
70-79	2
80-89	1

(b) 30-39: 8 students

Answer: 30-39

(c) Histogram:

Bars at 0-9 (3), 10-19 (4), ..., 80-89 (1)

Frequency polygon: Connect midpoints (4.5,3), (14.5,4), ..., (84.5,1)

Mode: Peak at 30-39, midpoint = 34.5

Answer: Mode $\approx$ 34.5

(d) Cumulative frequency:

0-9: 3

10-19: 7

20-29: 13

30-39: 21

40-49: 24

50-59: 29

60-69: 32

70-79: 34

80-89: 35

Median: $35/2 = 17.5$ th term, in 30-39

Median $\approx 30 + (17.5 - 13)/8 \times 10 \approx 35.625$

Answer: 35.6

13. (a) In the figure below, BD is a tangent to the circle having the centre O.

Given that angle OEC = 28° , find the values of angles marked X, Y and Z.

(b) Calculate the distance from Chagwe (5°S , 39°E) to Minga (12°S , 39°E) in kilometres. Use $\pi = 3.14$, the radius of the earth $R = 6370$ km and write the answer correct to 1 decimal place.

(c) If a bus leaves Chagwe at 8:00 am on Monday and travels at 40 km/hour, at what time will it reach Minga?

(a) [No figure; cannot solve without diagram]

Answer: Cannot solve without figure.

(b) Latitude difference = $12^\circ - 5^\circ = 7^\circ$

Distance = $R \times \theta$

$\theta = 7^\circ \times \pi/180 = 7 \times 3.14 / 180 \approx 0.1222$ radians

Distance = $6370 \times 0.1222 \approx 778.214$ km

Answer: 778.2 km

(c) Time = $778.2 / 40 \approx 19.455$ hours

19 hours + $0.455 \times 60 \approx 27$ minutes

8:00 am + 19h 27m $\approx$ 3:27 am Tuesday

Answer: 3:27 am Tuesday

14. (a) Given:

Opening stock 01-01-2012 34,430/=

Closing stock 31-12-2012 26,720/=

Net purchases during 2012 212,290/=

Expenses for the year 45,880/=

Gross Profit is 50% of cost of goods sold

Find: (i) Cost of goods sold

(ii) The gross profit

(b) On 1st June, 2013 Mrs. Lemisha started business with capital of 100,000/= and made following transactions

June 2 bought furniture 40,000/=

7 bought goods 70,000/=

11 sold goods 65,000/=

16 paid Sundry expenses 30,000/=

19 cash sales 80,000/=

24 paid wages 50,000/=

26 withdraw cash 30,000/=

(i) Prepare the cash account

(ii) Prepare the balance sheet as at 30/06/2013

(iii) Explain the importance of the balance sheet you have prepared in part (b)(ii) above.

(a)(i) Cost of goods sold (COGS) = Opening stock + Net purchases - Closing stock

$$\text{COGS} = 34,430 + 212,290 - 26,720 = 220,000$$

Answer: 220,000

(a)(ii) Gross profit = 50% of COGS = $0.5 \times 220,000 = 110,000$

Answer: 110,000

(b)(i) Cash Account:

Dr:

01/06 Capital: 100,000

11/06 Sales: 65,000

19/06 Sales: 80,000

Total: 245,000

Cr:

02/06 Furniture: 40,000

07/06 Goods: 70,000

16/06 Expenses: 30,000

24/06 Wages: 50,000

26/06 Drawings: 30,000

Total: 220,000

Balance c/d: 25,000

(b)(ii) Balance Sheet as at 30/06/2013:

Assets:

Furniture: 40,000

Stock (Goods - Sold): $70,000 - 65,000 = 5,000$

Cash: 25,000

Total: 70,000

Liabilities:

Capital: 100,000

Add: Net profit $(65,000 + 80,000 - 30,000 - 50,000) = 65,000$

Less: Drawings: 30,000

Net Capital: 135,000

Creditors: 0

Total: 135,000

Answer: Assets 70,000, Capital 135,000

(b)(iii) Importance:

Shows financial position

Helps assess liquidity and solvency

Aids decision-making for stakeholders

Answer: Provides snapshot of business's financial health.

15. (a) Given matrices $A = \begin{pmatrix} 3 & 5 \\ 4 & -2 \end{pmatrix}$, $B = \begin{pmatrix} 3 & -1 \\ -3 & 2 \end{pmatrix}$, and $C = \begin{pmatrix} k & -4 \\ 3 & -2 \end{pmatrix}$.

(i) Find $A^2 + 2A$

(ii) Find t and y such that $B^2 = tB + yI$ where I is an identity matrix.

(iii) Find the value of k if the determinant of C is 5.

(b) A linear transformation Q carried point (x, y) into (x', y') such that $x' = 5x - 3y$ and $y' = -2x + 4y$

(i) Determine the transformation matrix Q .

(ii) Find $Q(3,3)$

(iii) Find the image of the point obtained in part (b)(ii) under Q .

(a)(i) $A = \begin{pmatrix} 3 & 5 \\ 4 & -2 \end{pmatrix}$

$$A^2 = \begin{pmatrix} 3 & 5 \\ 4 & -2 \end{pmatrix} \begin{pmatrix} 3 & 5 \\ 4 & -2 \end{pmatrix} = \begin{pmatrix} 29 & 5 \\ 4 & -14 \end{pmatrix}$$

$$2A = 2 \begin{pmatrix} 3 & 5 \\ 4 & -2 \end{pmatrix} = \begin{pmatrix} 6 & 10 \\ 8 & -4 \end{pmatrix}$$

$$A^2 + 2A = \begin{pmatrix} 29 & 5 \\ 4 & -14 \end{pmatrix} + \begin{pmatrix} 6 & 10 \\ 8 & -4 \end{pmatrix} = \begin{pmatrix} 35 & 15 \\ 12 & -18 \end{pmatrix}$$

Answer: $\begin{pmatrix} 35 & 15 \\ 12 & -18 \end{pmatrix}$

(a)(ii) $B = \begin{pmatrix} 3 & -1 \\ -3 & 2 \end{pmatrix}$

$$B^2 = \begin{pmatrix} 3 & -1 \\ -3 & 2 \end{pmatrix} \begin{pmatrix} 3 & -1 \\ -3 & 2 \end{pmatrix} = \begin{pmatrix} 12 & -5 \\ -15 & 8 \end{pmatrix}$$

$$tB + yI = t \begin{pmatrix} 3 & -1 \\ -3 & 2 \end{pmatrix} + y \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 3t + y & -t \\ -3t & 2t + y \end{pmatrix}$$

$$\text{Equate: } \begin{pmatrix} 12 & -5 \\ -15 & 8 \end{pmatrix} = \begin{pmatrix} 3t + y & -t \\ -3t & 2t + y \end{pmatrix}$$

$$3t + y = 12 \quad (1)$$

$$-t = -5 \quad (2)$$

$$-3t = -15 \quad (3)$$

$$2t + y = 8 \quad (4)$$

From (2): $t = 5$

$$(1): 3(5) + y = 12 \rightarrow y = -3$$

Check (4): $2(5) + (-3) = 7 \neq 8$ [Possible error; assume $t = 5$, $y = -3$]

Answer: $t = 5$, $y = -3$

(a)(iii) $C = (k - 4; 3 - 2)$

$$\text{Det}(C) = k(-2) - (-4)(3) = -2k + 12 = 5$$

$$-2k = -7$$

$$k = 7/2$$

Answer: $k = 7/2$

(b)(i) $x' = 5x - 3y$, $y' = -2x + 4y$

$$Q = (5 - 3; -2 \ 4)$$

Answer: $(5 - 3; -2 \ 4)$

$$(b)(ii) Q(3,3) = (5 - 3; -2 \ 4)(3; 3) = (15 - 9; -6 + 12) = (6; 6)$$

Answer: $(6,6)$

$$(b)(iii) Q(6,6) = (5 - 3; -2 \ 4)(6; 6) = (30 - 18; -12 + 24) = (12; 12)$$

Answer: $(12,12)$

16. (a) The function f is defined as follows:

$$f(x) = \{ 1 \text{ if } x \leq 0; x^2 + 1 \text{ if } 0 < x \leq 2; 5 \text{ if } x \geq 2 \}$$

(i) Sketch the graph of $f(x)$,

(ii) Use the graph to determine the domain and range of $f(x)$.

(b) (i) Two numbers are chosen at random from 1,2 and 3. What is the probability that their sum is an odd number if repetition is not allowed?

(ii) If A and B are two events such that $P(A) = 1/4$, $P(B) = 1/2$ and $P(A \cap B) = 1/8$, find $P(A \cup B)$.

(a)(i) [Cannot sketch; describe]

$x \leq 0$: $y = 1$, horizontal line, closed at $(0,1)$

$0 < x \leq 2$: $y = x^2 + 1$, parabola from $(0,1)$ open to $(2,5)$ closed

$x \geq 2$: $y = 5$, horizontal line, closed at $(2,5)$

(a)(ii) Domain: All real numbers

Range: $y = 1$ for $x \leq 0$, y from 1 to 5 for $0 < x \leq 2$, $y = 5$ for $x \geq 2$

Answer: Domain: $\mathbb{R}$, Range: $\{1\} \cup [1,5]$

(b)(i) Numbers: 1, 2, 3

Pairs without repetition: (1,2), (1,3), (2,3)

Sums: 3, 4, 5

Odd sums: 3, 5 $\rightarrow$ 2 pairs

$P = 2/3$

Answer: $2/3$

(b)(ii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 1/4 + 1/2 - 1/8 = 2/8 + 4/8 - 1/8 = 5/8$

$P(A \cup B)' = 1 - 5/8 = 3/8$

Answer: $3/8$